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ABSTRACT

This instructor's guide consists of materials for use in teaching a course in algebra designed for students enrolled in postsecondary vocational and technical education programs. Addressed in the eight individual units of the guide are the following topics: signed numbers, algebraic notation, equations and inequalities, solving algebraic equations, binomials and polynomials, simultaneous equations, rectangular coordinates and linear equations, and quadratic equations. Each unit contains some or all of the following components: a unit objective, specific performance objectives, suggested activities for the instructor, a list of references, information sheets, transparency masters, assignment sheets, a test evaluation, and test answers. (MN)

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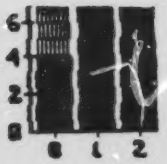

ALGEBRA

Instructor Edition

POST-SECONDARY

VOCATIONAL-TECHNICAL LEVEL

AREA OF VOCATIONAL EDUCATION
COLLEGE OF EDUCATION
UNIVERSITY OF ARKANSAS
FAYETTEVILLE, ARKANSAS 72701

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ALGEBRA

by

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Developed at the

UNIVERSITY OF ARKANSAS, FAYETTEVILLE

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in cooperation with the

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Appreciation is extended to those individuals who contributed their time and efforts to the development of this guide.

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Black River	Pulaski
Northwest	Rich Mountain
Pines	White River

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USE OF PUBLICATION

Instructional Units

This guide contains 8 units. Each instructional unit includes some or all of the basic components of a unit of instruction: performance objectives, suggested activities for teachers, information sheets, assignment sheets, visual aids, tests, and answers to the test. Units are planned for more than one lesson or class period of instruction.

Careful study of each instructional unit by the teacher will help to determine:

A. The amount of material that can be covered in each class period

B. The skills which must be demonstrated
   1. Supplies needed
   2. Equipment needed
   3. Amount of practice needed
   4. Amount of class time needed for demonstrations

C. Supplementary materials such as pamphlets or filmstrips that must be ordered

D. Resource people who must be contacted

Objectives

Each unit of instruction is based on performance objectives. These objectives state the goals of the course, thus providing a sense of direction and accomplishment for the student.

Performance objectives are stated in two forms: unit objectives, stating the subject matter to be covered in a unit of instruction; and specific objectives, stating the student performance necessary to reach the unit objective.

Since the objectives of the unit provide direction for the teaching-learning process, it is important for the teacher and students to have a common understanding of the intent of the objectives. A limited number of performance terms have been used in the objectives for this curriculum to assist in promoting the effectiveness of the communication among all individuals using the materials.

Reading of the objectives by the student should be followed by a class discussion to answer any questions concerning performance requirements for each instructional unit.

Teachers should feel free to add objectives which will fit the material to the needs of the students and community. When teachers add objectives, they should remember to supply the needed information, assignment and/or job sheets, and criterion tests.

Suggested Activities for the Instructors

Each unit of instruction has a suggested activities sheet outlining steps to follow in accomplishing specific objectives. Duties of instructors will vary according to the particular unit; however, for best use of the material they should include the following: provide students with objective sheet, information sheet, and assignment sheets; make transparencies and arrange for resource materials and people; discuss unit and specific objectives and information sheet; give test. Teachers are encouraged to use any additional instructional activities and teaching methods to aid students in accomplishing the objectives.

Information Sheets

Information sheets provide content essential for meeting the cognitive(knowledge) objectives in the unit. The teacher will find that the information sheets serve as an excellent guide for presenting the background knowledge necessary to develop the skill specified in the unit objective.

Students should read the information sheets before the information is discussed in class. Students may take additional notes on the information sheets.

Transparency Masters

Transparency masters provide information in a special way. The students may see as well as hear the materials being presented, thus reinforcing the learning process. Transparencies may present new information or they may reinforce information presented in the information sheets. They are particularly effective when identification is necessary.

Transparencies should be made and placed in the notebook where they will be immediately available for use. Transparencies direct the class's attention to the topic of discussion. They should be left on the screen only when topics shown are under discussion.

Assignment Sheets

Assignment sheets give direction to study and furnish practice for paper and pencil activities to develop the knowledges which are necessary prerequisites to skill development. These may be given to the student for completion in class or used for homework assignments. Answer sheets are provided which may be used by the student and/or teacher for checking student progress.

Test Evaluation

Paper-pencil and performance tests have been constructed to measure student achievement of each objective listed in the unit of instruction. Individual test items may be pulled out and used as a short test to determine student achievement of a particular objective. This kind of testing may be used as a daily quiz and will help the teacher spot difficulties being encountered by students in their efforts to accomplish the unit objective. Test items for objectives added by the teacher should be constructed and added to the test.

Test Answers

Test answers are provided for each unit. These may be used by the teacher and/or student for checking student achievement of the objectives.

SIGNED NUMBERS

UNIT I

UNIT OBJECTIVE

After completion of this unit, the student should be able to combine positive and negative numbers. This knowledge will be evidenced by correctly performing the procedures outlined in the assignment sheets and by successfully scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Add positive and negative numbers.
2. Subtract positive and negative numbers.
3. Multiply and divide positive and negative numbers.

SIGNED NUMBERS

UNIT I

SUGGESTED ACTIVITIES

I. Make transparency.

II. Provide student with objective sheet.

III. Provide student with information and assignment sheets.

IV. Discuss unit and specific objectives.

V. Discuss information and assignment sheets.

VI. Illustrate transaparency.

VII. Assist students as they work through assignment sheets.

VIII. Give test.

IX. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective sheet

B. Information sheet

C. Transparency master #1--Number Line

D. Assignment sheets-

1. Assignment Sheet #1--Adding Positive and Negative Numbers

2. Assignment Sheet #2--Subtracting Positive and Negative Numbers

3. Assignment Sheet #3--Multiplying and Dividing Positive and Negative Numbers

E. Answers to Assignment Sheets

F. Test

G. Answers to Test

II. References

A. Chant, Alfred E., Barbara J. Montalto and Barbara A. Enfinger. Reaching Math Competence New York, NY: Globe Book Company, Inc., Cambridge Division, 1981.

B. Kerr, Donald J., Jr. Elementary Algebra New York, NY: McGraw-Hill, 1980.

C. Olivo, C. Thomas and Thomas P. Olivo. Basic Mathematics Simplified 4th Edition. Albany, NY: Delmar Publishers Incorporated, 1977.

D. Shea, James T. Algebra Austin, TX: Steck-Vaughn Company, 1975.

E. Smith, Robert D. Vocational-Technical Mathematics Albany, NY: Delmar Publishers Incorporated, 1983.

F. Wiltsie, David. Practical Uses for Formulas and Equations Bishop, CA: Motivation Development Incorporated, 1982.

SIGNED NUMBERS
UNIT I

INFORMATION SHEET

I. Terms and Definitions

A. Signed Numbers--Numbers that have attached to them either a positive, or plus (+) sign or a negative, or minus (-) sign

B. Positive Numbers--Numbers that are greater than zero

C. Negative Numbers--Numbers that are less than zero

(NOTE: Any number without a sign is assumed to be positive. Zero is neither positive nor negative. It has no sign.)

II. Procedures

A. Adding numbers with the same sign--Add the numbers and give the answer the common sign.

Examples:

$$\begin{array}{rr} & +15 \\ + & +\ 5 \\ \hline & +20 \end{array} \qquad \begin{array}{rr} & -15 \\ + & -\ 5 \\ \hline & -20 \end{array}$$

B. Adding numbers with different signs--Find the difference between the two numbers and give the answer the sign of the higher number.

Examples:

$$\begin{array}{rr} & +15 \\ + & -\ 2 \\ \hline & +13 \end{array} \qquad \begin{array}{rr} & -18 \\ + & +\ 3 \\ \hline & -15 \end{array}$$

C. Adding more than two signed numbers--First add all the positive numbers, then add all the negative numbers, and finally find the sum of these two numbers respectively.

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Example:

-12			
+ 8			
- 9	-12		
- 6	- 9	+ 8	-27
+ 3	- 6	+ 3	+11
	-27	+11	-16 (answer)

D. Subtracting signed numbers--Change the sign of the subtrahend (bottom number) to the opposite sign, and proceed as with addition.

Examples:

```
  +15
- - 5
------
  +20
```

Change the -5 to +5 and add the numbers because both of the numbers are now the same. The answer gets the common sign.

```
  -15
- - 5
------
  -10
```

Change the -5 to +5 and subtract because the two numbers now have different signs. The answer gets the sign of the higher number.

E. Multiplication and Division of Signed Numbers--Perform the indication operation (multiply or divide the numbers). Give the answer the positive (+) sign if the numbers in the problem have the same sign. Give the answer a negative sign (-) if the numbers in the problem have different signs.

Examples:

(-5) (-2) = +10 (+10) (-2) = -20

(NOTE: The procedures for multiplying and dividing signed numbers are the same.)

F. Multiplying a series of signed numbers--Do the multiplication first. Then count the number or negative signs in the problem. If there is an odd number of negative signs, the answer is negative. If there is an even number of negative signs, the answer is positive.

Example: Find the produce (multiply)

(+2) (-3) (-4) (+5) =

2 x 3 x 4 x 5 = 120

Because there is an even number of negative signs (2), the answer is positive.

The answer is +120.

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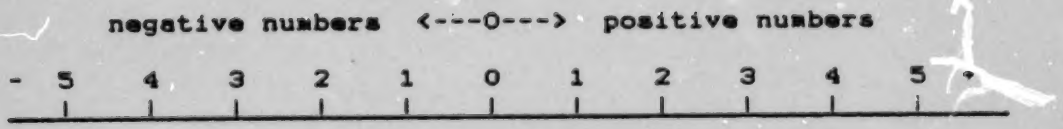

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SIGNED NUMBERS
UNIT I

ASSIGNMENT SHEET #1--ADDING SIGNED NUMBERS

Directions: Add the following problems. Be sure to apply the rules for adding signed numbers.

a. -12
$\quad -3$
$\quad -6$
$\overline{\qquad}$

b. $+11$
$\quad -17$
$\overline{\qquad}$

c. -3 and -13 =

d. -6
$\quad +4$
$\quad +7$
$\overline{\qquad}$

e. -14
$\quad -13$
$\quad +16$
$\overline{\qquad}$

f. -35
$\quad +18$
$\quad -11$
$\overline{\qquad}$

g. -17
$\quad -23$
$\quad -19$
$\overline{\qquad}$

h. -9 and $+8$ and $+18$ and -16 =

i. $(-8) + (35) + (-16) + (-13)$ =

j. $(-4) + (-5) + (+28) + (-15)$ =

SIGNED NUMBERS
UNIT I

ASSIGNMENT SHEET #2--SUBTRACTING SIGNED NUMBERS

Directions: Subtract the following problems. Be sure to apply the rules for subtracting signed numbers.

a. $+12 - (+15) =$

b. $-16 - (-9) =$

c. $+23 - (-31) =$

d. $-63 - (+29) =$

e. $+9.3 - (-7.2) =$

f. $-2\ 1/4 - (-7\ 1/4) =$

g. $+4\ 1/4 - (+9\ 7/8) =$

h. $(-12) - (-12) =$

i. $(+20) - (-15) =$

j. $(-17) - (+16) =$

SIGNED NUMBERS
UNIT I

ASSIGNMENT SHEET #3--MULTIPLYING AND DIVIDING SIGNED NUMBERS

Directions: Multiply and divide the following problems as indicated. Remember to apply the rules of multiplying and dividing signed numbers.

a. $(-4)\ (+9) =$

b. $(+14)\ (-3) =$

c. $(-23)\ (-5) =$

d. $(+8)\ (-8) =$

e. $(-4)\ (0) =$

f. $\dfrac{+5}{+1} =$

g. $\dfrac{+12}{-3} =$

h. $\dfrac{-25}{+10} =$

i. $\dfrac{-6}{-3} =$

j. $\dfrac{(+8)\ (-4)}{-16} =$

k. $\dfrac{50}{(-2)(+5)} =$

l. $\dfrac{(-6)\ (-6)}{12} =$

m. $\dfrac{(-2)(+8)(-12)}{(-6)} =$

n. $\dfrac{(-1)(+2)(+5)(-7)}{(-10)(+2)(-14)(-4)} =$

o. $\dfrac{+3/4}{-2/3} =$

SIGNED NUMBERS
UNIT I

ANSWERS TO ASSIGNMENT SHEETS

ASSIGNMENT SHEET #1

a. -21
b. -6
c. -16
d. -11
e. -28
f. -28
g. -69
h. + 1
i. - 2
j. + 4

ASSIGNMENT SHEET #2

a. - 3
b. - 7
c. +54
d. -92
e. +16.5
f. + 5
g. - 5 5/8
h. 0
i. +35
j. -33

ASSIGNMENT SHEET #3

a. -36
b. +42
c. +115
d. -64
e. 0
f. + 5
g. - 4
h. - 2.5
i. + 2
j. + 2
k. - 5
l. +3
m. -32
n. - 1/16
o. - 1 1/8

SIGNED NUMBERS
UNIT I

TEST

Directions: Solve the following problems. Perform the operation indicated. Remember to apply the rules of signed numbers.

1. Add:

a. $+13 + (-5)$

b. $+27 + (+3)$

c. $-27 + (-3)$

d. $-43 + (+2)$

e. $-2 + (+7)$

f. $-\frac{1}{3} + (+\frac{1}{7})$

g. $+14 + (-21)$

h. $-29 + (-45)$

2. Subtract:

a. $+29 - (+48)$

b. $+\frac{9}{8} - (-\frac{4}{3})$

e. $+7 - (-7)$

f. $-125 - (-237)$

c. $\begin{array}{r} -16 \\ -25 \\ \hline \end{array}$

g. $\begin{array}{r} +\ 9/2 \\ -\ 3/2 \\ \hline \end{array}$

d. $\begin{array}{r} -\ 7/8 \\ -\ 3/8 \\ \hline \end{array}$

h. $\begin{array}{r} -\ 7/6 \\ -\ 1/3 \\ \hline \end{array}$

3. Multiply or divide as indicated:

a. $(-3)\ (+5) =$

b. $(-\ 3/8)\ (-\ 2/3) =$

c. $(+25)\ (-14) =$

d. $(-3)\ (+5) =$

e. $(-48)\ (-62) =$

f. $\dfrac{-12}{-2} =$

g. $\dfrac{+\ 9}{-\ 3} =$

h. $\dfrac{-\ 14}{+\ 2} =$

i. $\dfrac{-\ 3}{-\ 1} =$

SIGNED NUMBERS
UNIT I

ANSWERS TO TEST

1. a. +8
   b. +30
   c. -30
   d. -41
   e. +5
   f. - 4/12
   g. -7
   h. -74

2. a. +77
   b. + 2 11/14
   c. +9
   d. - 1/2
   e. +14
   f. +112
   g. +6
   h. - 5/6

3. a. -72
   b. + 1/4
   c. -350
   d. -15
   e. +2,976
   f. +6
   g. -2
   h. -7
   i. +3

ALGEBRAIC NOTATION
UNIT II

UNIT OBJECTIVE

After completion of this unit, the student should be able to interpret algebraic notation. This knowledge will be evidenced by correctly performing the procedures outlined in the assignment sheets and by successfully scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Identify given terms used in algebra.
2. Write algebraic expressions.

ALGEBRAIC NOTATION
UNIT II

SUGGESTED ACTIVITIES

I. Provide student with objective sheet.

II. Provide student with information and assignment sheets.

III. Discuss unit and specific objectives.

IV. Discuss information and assignment sheets.

V. Assist students as they work through assignment sheets.

VI. Give test.

VII. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective sheet

B. Information sheet

C. Assignment sheets-

1. Assignment Sheet #1--Identifying Terms Used in Algebra

2. Assignment Sheet #2--Writing Algebraic Expressions

D. Answers to Assignment Sheets

E. Test

F. Answers to test

II. References

A. Chant, Alfred E., Barbara J. Montalto and Barbara A. Enfinger. Reaching Math Competence New York, NY: Globe Book Company, Inc., Cambridge Division, 1981.

B. Kerr, Donald J., Jr. Elementry Algebra New York, NY: McGraw-Hill, 1980.

C. Olivo, C. Thomas and Thomas ?. Olivo. Basic Mathematics Simplified, 4th Edition. Albany, NY: Delmar Publishers Incorporated, 1977.

D. Shea, James T. Algebra Austin, TX: Steck-Vaughn Company, 1975.

E. Smith, Robert D. Vocational-Technical Mathematics Albany, NY: Delmar Publishers Incorporated, 1983.

F. Wiltsie, David. Practical Uses for Formulas and Equations Bishop, CA: Motivation Development Incorporated, 1982.

ALGEBRAIC NOTATION
UNIT II

INFORMATION SHEET

I. Terms and Defintions

(NOTE: An understanding of the "language" of algebra is essential to an understanding of algebra.)

A. Mathematical expression--A single number or a group of numbers to be combined by arithmetic

Example: $23 + 5$

B. Equation--A relationship expressed mathematically in which the expression on one side of an equals sign, =, has an equal value to the expression on the opposite side of the equals sign

Example: $10 = 2 + 8$

C. Algebra--The process of solving for an unknown value in an expression

Example: $n = 2 + 5$
$n = 7$

D. Algebraic expression--Letters and numbers used to express a value

Example: $3 + n = 5$

E. Variable--Letters which represent values that can vary; also called literal numbers

Examples: b, x, y, s, etc.

F. Constant--The symbol which represents a definite numerical value; also called arithmetic numbers

Examples: 5, 16, 1/2, etc.

G. Term--One or more values (constants and/or variables) that stand for one number

Examples: 3, x, 4x, $3xy(a+b)$

(NOTE: Each of the four above examples can be called a term.)

H. Factor--Two or more numbers that are multiplied or divided in a term

Example: 30 has 2, 3, and 5 as factors;
$2 \times 3 \times 5 = 30$

(NOTE: Letters as well as numbers can be called factors.)

Example: $24 = 2ab$

(NOTE: In the example above, 2, a, and b are factors of the term, 24.)

I. Exponential Notation--A system for indicating that a value is to be multiplied a given number of times

J. Exponent or power--A superscript number which tells how many times a value is to be used as a factor

Example:

6^2

(NOTE: The above example reads 6 to the third power. The superscript 3 indicates that the 6 is to be used as a factor three times; that is, $6 \times 6 \times 6$

K. Root or base--The value that is to be multiplied by itself

(NOTE: A root may be a constant or a variable.)

6^2; X^2

1. Square Root--The number that is multiplied by itself to give the number inside the square root sign, $\sqrt{}$ the opposite of a number to the second power

Example:

$\sqrt{16} = 4$

II. Procedures

A. Determining powers

1. Determine which number is the base and which number the exponent

Example: 6^2

(NOTE: In the previous example, 6 is the base and 2 is the exponent.)

2. Write the base the number of times expressed by the exponent and multiply.

Example:

$$6^2 = 6 \times 6 = 36$$

B. Determining perfect square roots

1. Determine what number times itself would equal the number inside the $\sqrt{}$ sign

Example: $\sqrt{36}$ Ask: What number times itself equals 36?

2. Check your answer by multiplying it by itself the number of times indicated in the exponent

(NOTE: If you answered 6 in the above example, check by multiplying 6 x 6 = 36.)

3. Common square roots:

$\sqrt{1} = 1$		$\sqrt{121} = 11$	
$\sqrt{4} = 2$		$\sqrt{144} = 12$	
$\sqrt{9} = 3$		$\sqrt{169} = 13$	
$\sqrt{16} = 4$		$\sqrt{225} = 15$	
$\sqrt{25} = 5$		$\sqrt{400} = 20$	
$\sqrt{36} = 6$		$\sqrt{900} = 30$	
$\sqrt{49} = 7$		$\sqrt{1,600} = 40$	
$\sqrt{64} = 8$		$\sqrt{2,500} = 50$	
$\sqrt{81} = 9$		$\sqrt{3,600} = 60$	
$\sqrt{100} = 10$		$\sqrt{4,900} = 70$	

$\sqrt{6,400} = 80$ $\sqrt{10,000} = 100$

$\sqrt{8,100} = 90$

C. Writing Algebraic Expressions

1. Determine the operation(s) which should be used.

(NOTE: The following is a review of operations and the terms which indicate each operation.)

a. Addition

sum: The sum of a number and 4; $n + 4$
total: The total of 8 and a number; $8 + x$
increased by: A number increased by 20; $x + 20$
plus: 15 plus a number; $15 + n$
more than: 12 more than a number; $x + 12$

b. Subtraction

less than: 4 less than a number; $n - 4$
decreased by: 10 decreased by a number; $10 - n$
subtracted from: A number subtracted from 50; $50 - n$; 120 minus a number; $120 - n$

c. Multiplication

product: The product of 5 and a number; $5x$
times: 2 times a number; $2n$
a fraction of: 3/4 of a number; $3/4\ n$

(NOTE: The symbol x is not used to indicated multiplication in algebra because x is often used as a variable. Instead, multiplication is indicated by the raised dot, $\cdot$, $2 \cdot 3$, by parentheses $(2)(3)$, or by a constant and/or variable together, $2xy$.)

d. Division

divided by; a number divided by 13; $x/13$
16 divided by a number; $16/n$

2. Write the expression as a number sentence.

ALGEBRAIC NOTATION
UNIT II

ASSIGNMENT SHEET #1--IDENTIFY TERMS USED IN ALGEBRA

1. In the following, circle the variables:

$$6, \quad 3x, \quad 2ab, \quad \frac{3(2+x)}{y}, \quad d = rt$$

2. In the following, circle the constants:

$$6, \quad 3x, \quad 2ab, \quad \frac{3(2+x)}{y}, \quad d = rt$$

3. In the following, circle each term:

$$5 + n, \quad 3xy, \quad 2x + 3, \quad \frac{5x}{5}, \quad 2x - 3y$$

4. How many terms are there in the following algebraic expression?

$$x + 4xyz - \frac{6p}{y + 2} - 8$$

5. Determine the square root of the following:

$\sqrt{16}$ = $\sqrt{81}$ = $\sqrt{100}$ =

6. Illustrate the following values by use of exponents:

a. 81 =
b. 100 =
c. 16 =
d. 216 =
e. 49 =
f. 27 =

ALGEBRAIC NOTATIONS
UNIT II

ASSIGNMENT SHEET #2--WRITING ALGEBRAIC EXPRESSIONS

Directions: Write the following phrases as algebraic expressions. Represent the unknown with X.

1. The sum of 5 times a number and 3

2. The product of 6 and a number

3. 8 subtracted from 4 times a number

4. 9 times a number decreased by 2

5. A number increased by 6

6. A number plus 3 divided by 2

7. 6 times a number multiplied by 6

8. The difference between a number and 36

9. The product of 5 times a number

10. Twice a number decreased by 2

ALGEBRAIC NOTATION
UNIT II

ANSWERS TO ASSIGNMENT SHEETS

Answer Sheet #1

1. 6, $3x$, $2ab$, $\frac{3(a+x)}{y}$, $d = rt$

2. 6, $3x$, $2ab$, $\frac{3(a+x)}{y}$, $d = rt$

3. $5 + n$, $3xy$, $2x + 3$, $\frac{5x}{5}$, $2x - 3y$

4. 4

5. 4, 9, 10

6. a. 9^2
   b. 10^2
   c. 4^2
   d. 6^3
   e. 7^2
   f. 3^3

Assignment Sheet #2

1. $5x + 3$
2. $6x$
3. $4x - 8$
4. $9x - 2$
5. $x + 6$
6. $\frac{x + 3}{2}$
7. 6^{2x}
8. $x - 36$
9. $5x$
10. $2x - 2$

ALGEBRAIC NOTATION
UNIT II

ANSWERS TO TEST

1. a. $3abc + \dfrac{5x + 2}{2a + xyz}$

   b. $5w = x + 2y - 3z$

   c. 2^4, 6^2, 2^3

   d. 4, 12

2. a. $3x + 5$

   b. $x + x + 1$ or $2x + 1$

   c. $(x)\ (x-5)$

   d. $152 - n$

   e. $4x - 3x$

   f. $x + 2$

   g. $\dfrac{x}{6}$

   h. $2x + x$

   i. $\dfrac{4x}{2}$

EQUATIONS AND INEQUALITIES
UNIT III

UNIT OBJECTIVE

After completion of this unit, the student should be able to represent relationships with algebraic expressions. This knowledge will be evidenced by correctly performing the procedures outlined in the assignment sheets and by successfully scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Combine and simplify monomials.
2. Write simple equations.
3. Illustrate inequalities.
4. Express relationships with ratio and proportion.
5. Employ correct order of operations.

EQUATIONS AND INEQUALITIES
UNIT II

SUGGESTED ACTIVITIES

I. Provide student with objective sheet.

II. Provide student with information and assignment sheets.

III. Discuss unit and specific objectives.

IV. Discuss information and assignment sheets.

V. Assist students as they work through assignment sheets.

VI. Give test.

VII. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective sheet

B. Information sheet

C. Assignment sheets-

1. Assignment Sheet #1--Combine and Simplify Monomials

2. Assignment Sheet #2--Write Simple Equations

3. Assignment Sheet #3--Illustrate Inequalities

4. Assignment Sheet #4-- Express Relationships with Ratio and Proportion

D. Answers to Assignment Sheets

E. Test

F. Answers to test

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EQUATIONS AND INEQUALITIES
UNIT III

INFORMATION SHEET

I. Terms and Definitions

A. Like terms--Terms which a "alike"; that is, can be combined

Example: 2 ft. and 3 ft. are like terms;
2x and 3 x are like terms

B. Unlike terms--Terms that cannot be combined except by the + sign.

Example: 2 ft. 1 in. cannot be combined unless the larger unit is converted to its equivalent in the smaller unit; 2 ft. 1 in. = 25 in. or as a fraction, 2 1/12 ft.; 2x, 2 y cannot be combined except by the + sign, 2x + 2y

C. Monomial--An expression of a single term or quantity

Examples: a, 2a, 1/2a. ab, lw

D. Binomial--An expression of two terms

Example: 2a + b

E. Equation--The symbolic statement that two expressions are equal

F. Inequalities--The symbolic statement that two expressions are <u>not</u> equal

(NOTE: Inequality symbols read from left to right.)

(NOTE: The diagonal line negates the basic = sign.)

1. $\neq$ means "is not equal to"
2. $<$ means "is less than"
3. $\leq$ means "is less than or equal to"
4. $>$ means "is greater than"
5. $\geq$ means "is greater than or equal to"
6. $\not<$ means "is not less than is is not equal to"

7. $\not>$ means "is not greater than and is not equal to"

G. Ratio--A comparison of quantities of the same kind

Example: The ratio of 1 (16 oz.) pound to 1 ounce is 16 to 1

H. Proportion--Two equal ratios

Example: 40:30 or 40/30

I. Exponent--The superscript number which tells how many times a value is to be used as a factor

J. Coefficient--The numerical factor

II. Combining Monomials--Like terms may be combined by addition, subtractions, multiplication or division.

A. Adding Monomials--Add the coefficients and affix the literal factors

Example: Add

$13y + 2y = 15y$

(NOTE: Remember that a literal term without a number in front of it is assumed to have a 1 before it.)

Example: Add

$5x + x = 6x$

(NOTE: Remember to apply the rules of signed numbers when combining monomials.)

Example: Add

$$\begin{array}{r} -3abc \\ +2abc \\ \hline abc \end{array}$$

B. Subtracting Monomials--Change the sign of the subtrahend and add

Examples: Subtract

7a	18ab	-14xyz
2a	- 6ab	5xyz
5a	24ab	- 9xyz

C. Multiplying Monomials--Multiply the coefficients and affix the literal factors

Examples:

5x	-5abc	.7y	(1/2x)(4) = 2x
4	-3	-3	
20 x	15abx	-2.1y	

D. Dividing Monomials--Divide the coefficients and affix the literal factors

Examples: $\frac{20a}{4} = 5a$ $\qquad \frac{22xyz}{-11} = -2xyz$

E. Adding and Subtracting with Exponents--Add the coefficients and affix the literal factor

Examples:

Add	Subtract
$2x^2$	$6XY^2$
$3X^2$	$2XY^2$
------	------
$5X^2$	$4XY^2$

F. Multiplication with Exponents--Multiply the coefficients and <u>add</u> the exponents

Examples:

$(x)(x) = x^2$

$(b^3)(b) = b^4$

(NOTE: A variable without an exponent symbol is assumed to have an exponent of 1.)

Example:

$$\begin{array}{r} ax \\ ax \\ \hline (a)(a)(x)(x) = a^2x^2 \end{array}$$

G. Division with Exponents--Divide the coefficients and subtract the exponents.

Examples:

$$\frac{x^3}{x} = x^2 \qquad \frac{16a^2}{4a} = 4a$$

III. Equations

A. Writing Equations--Writing an equation is similar to writing expressions. However, an equation represents two algebraic expressions which are equal.

(NOTE: Watch for key words in equations such as equals and is. Use the symbol = for these words in the equation.)

Example: The sum of a number and 15 is 36.

$$n + 15 = 36$$

IV. Inequalities--Algebraic representations of expressions which are not equal are made by use of inequality symbols. Inequalities are written by use of the appropriate symbol to demonstrate the true relationship between two expressions.

Examples: $x - 5 \leq 6$ reads a number minus 5 is less than or equal to 6

$x - 5 < 6$ reads a number minus 5 is less than 6

$x - 5 > 6$ reads a number minus 5 is greater than 6

V. Ratios--Comparisons of quantities of the same kind may be written two ways.

Examples: a comparison of 1 to 12 may be written as 1:12 or as 1/12

(NOTE: Ratios may be reduced just as a fraction can be reduced to lowest terms.)

Example: 15:40 or 15/40 may be written 3:8 or 3/8

VI. Proportions--Two equal ratios are called a proportion, which may be written two ways.

(NOTE: The first and last terms are called extremes; the second and third terms are called means.)

Example: 3/4 = 9/12 or 3:4 = 9:12

(NOTE: The "cross-products" of equations are equal.)

Example: 3/4 = 9/12

3x12 = 4x9 or 36 = 36

(NOTE: The fact that the cross products are equal is very useful in finding an unknown quantity when the other three quantities are known.)

Example:

An 8 ft. tall pole is related to a tree's height in the same way as the pole's 6 ft. shadow is related to the tree's 30 ft. shadow. This relationship can be represented by the proportion:

8:x = 6:30 when x represents the height of the tree

(NOTE: The order of the comparisons must remain the same in each ratio of the proportion. In the above example, the pole's height compares to the tree's height in the same way that the pole's shadow compares to the tree's shadow. This means (8)(30) = 6x or 240/6 = 40.)

VII. Order of Operations--When there is more than one operation, the following order should be observed:

A. Do operations inside parentheses or above or below fraction bars first.

B. Do powers and roots second.

C. Multiply and divide third.

D. Add and subtract last.

Example: (13 +2) - (2)(3) = (15)-(6) = 9

VII. Parentheses

A. Parentheses are used as symbols of inclusion and are useful for indicating operations which are to be done first; all quantities within a parenthesis are treated as one term until the parenthesis is removed.

Example:

$$\frac{(6 + 3)}{x} \quad \text{means} \quad \frac{9}{x}$$

B. When the parenthesis is preceeded by a negative sign, all signs within the parenthesis are changed to the opposite sign. Operations within the parenthesis are then performed before the parenthesis is removed.

Example:

$$12 + 3 - (-6 + 2) = 15 + 4 = 19$$

(NOTE: When a parenthesis is preceeded by a positive sign, no change is made in the signs of the quantities removed from the parenthesis.)

Example:

$$45 - 2 + (3 + 5) = 43 + 8 = 51$$

C. When a coefficient is attached to the parenthesis, 2(3+6), each quantity within the parenthesis is to be multiplied by the coefficient.

Example:

2(3+6) means 2 times 3 + 2 times 6 = 18

EQUATIONS AND INEQUALITIES
UNIT III

ASSIGNMENT SHEET #1--COMBINE AND SIMPLIFY MONOMIALS

Directions: Combine the following by performing the indicated operation.

1. Add:

   a. $3x + 2y + 3 + x + 2y$

   b. $3bc + 3a - 2a$

   c. $1/2\ a - 1/4\ a + 5a$

   d. $28xy + 20xy + 3xy$

   e. $15a^2 + 5a^2 - 7a^2$

2. Subtract:

   a. $12a^2b$ / $3a^2b$

   b. $-14a$ / $- 5a$

   c. $5a$ / $- 2a$

   d. $- 17x^2$ / $5x^2$

   e. $+ 6a^3$ / $- 2a^3$

3. Multiply

   a. $5x$ / 2

   b. $3x^2$ / x^2

   c. $2m^2n^2$ / -7

   d. $3b^2$ / $4b$

   e. $- 1/2\ b$ / 4

4. Divide:

a. $\frac{15abc}{5} =$

b. $\frac{8y}{-4} =$

c. $\frac{25a^2}{5a} =$

d. $\frac{3x^4y^4}{xy} =$

e. $\frac{a^2b^3c^5}{ab^2c} =$

EQUATIONS AND INEQUALITIES
UNIT III

ASSIGNMENT SHEET #2--WRITE SIMPLE EQUATIONS

Directions: Write equations to represent the following. Let X represent the unknown.

1. The sum of two numbers is twenty-one. One of the numbers is seven more than the other.

2. A total of twelve is equal to three times a number.

3. A number is equal to two times the sum of five plus three.

4. One-third of a number is fifteen.

5. A number divided by three equals three hundred.

6. Three consecutive numbers equal one hundred fifty-six.

7. A number decreased by four times three is eighty-eight.

8. A number times the same number equals sixteen.

9. A number decreased by five equals sixty divided by two.

10. Two times a number equals thirteen plus seven.

EQUATIONS AND INEQUALITIES
UNIT III

ASSIGNMENT SHEET #3--ILLUSTRATE INEQUALITIES

Directions: Illustrate the following unequal relationships using the appropriate inequality symbol. Let X represent the unknown number.

1. A number is greater than 15

2. A number plus 5 is not equal to 33

3. A number divided by 15 is not equal to and is not greater than 45

4. Fifteen minus a number is less than 10

5. Twenty-five divided by a number is less than or equal to five

6. A number times 5 is not greater than and is not equal to twenty-five

7. A number is greater than fifty-nine

8. A number plus five is less than 66

9. Five times a number plus 3 is not less than and is not equal to 18

10. Thirty-three is less than a number plus 1

EQUATIONS AND INEQUALITIES
UNIT III

ASSIGNMENT SHEET #4--EXPRESS RELATIONSHIPS
WITH RATIO AND PROPORTION

Directions: Write a ratio to express the following relationships.

1. The drug dosage had one part drug to 6 parts water. Express this relationship as a ratio.

2. An automobile got forty-three miles to a gallon of gas. Write a ratio to express this miles per gallon relationship.

Write proportions to express these relationships:

3. John worked twice as fast as George. John finished six boards while George finished three.

4. A machine produces 280 pieces in 3.5 hours. How long does it take to produce 720?

5. If a tree casts a 21 ft. shadow, and a 4 ft. tall boy casts a 2 ft. shadow, how tall is the tree?

6. A machine uses 3 gallons of gasoline when it runs 3 1/2 hrs. How much will it use if it runs 7 hrs. at the same speed?

7. A metal joint 16 ft. long requires 80 rivets. How many rivets are required for a joint 11 ft. long?

8. If 15 men can build a wall 12 ft. high, how many men will be required to build a wall 20 ft. high in the same period of time if they work at the same rate?

9. If a road drops 5 feet in every 40 feet of distance, how much will it drop in 100 ft. of distance?

10. If you spent $160 during the first 4 days of a vacation, how much will you spend the following 3 days if your spending rate remains the same?

EQUATIONS AND INEQUALITIES
UNIT III

ASSIGNMENT SHEET #5--EMPLOY CORRECT ORDER OF OPERATIONS

Directions: Combine the following terms by employing the correct order of operations.

1. $3a \div b \div (2a \div 2b)$

2. $3a \div b - (2a - 2b)$

3. $\dfrac{2a \div 3b}{2a} \div 10a$

4. $a \div 4 \div (a-3)$

5. $\dfrac{a^3b^3c^3}{abc} \div abc$

6. $(15a)\ (3) \div 2a \div 6$

7. $\dfrac{25a^2b^3}{5} \div 3ab$

8. $a \div 7 - (2a \div 5)$

9. $(a \div b \div a \div b) \div (a^2 \div b^2)$

10. $\dfrac{225x^2y}{5} \div x\ (xy^3)$

EQUATIONS AND INEQUALITIES
UNIT III

ANSWERS TO ASSIGNMENT SHEETS

Assignment Sheet #1

1. a. $4x + 4y + 3$
   b. $3bc + a$
   c. $5\ 1/4\ a$
   d. $51\ xy$
   e. $13a^2$

2. a. $9a^2b$
   b. $9a$
   c. $7a$
   d. $-22x^3$
   e. $8a^3$

3. a. $10x$
   b. $3x^4$
   c. $-14m^2n^2$
   d. $12d^3$
   e. $-2b$

4. a. $3abc$
   b. $-2y$
   c. $5a$
   d. $3x^3y^3$
   e. abc^4

Assignment Sheet #2

1. $x + x + 7 = 21$
   $2x + 7 = 21$
2. $3x = 12$
3. $x = 2(5+3)$
4. $1/3x = 15$
5. $\frac{x}{3} = 300$
6. let x = 1st number
   let x+1 = 2nd number
   let x+2 = 3rd number
   $x + x+1 + x+2 = 156$
   $3x + 3 = 156$
7. $x - (4\ 3) = 88$
8. $x^2 = 16$
9. $x - 5 = \frac{60}{2}$
10. $2x = 13 + 7$

Assignment Sheet #3

1. $x > 5$
2. $x + 5 = 33$
3. $\frac{y}{15} \leq 45$
4. $15 - x < 10$
5. $\frac{25}{x} \leq 5$
6. $5x \geq 25$
7. $x > 59$
8. $x + 5 < 66$
9. $5x + 3 \leq 18$
10. $33 < x + 1$

Assignment Sheet #4

1. 1:6 or 1/6
2. 43:1 or 43/1
3. J:G = 2:1 = 6:3 or J/G = 2/1 = 6/3
4. 280:720 = 3.5:X or 280/720 = 3.5/X
5. x:4 = 21:2 or x/4 = 21/2
6. 3:x = 3 1/2:7 or 3/x = 3 1/2 / 7
7. 16:11 = 80:x or 16/11 = 80/x
8. 15:x = 12:20 or 15/x = 12/20

9. $5:x = 40:100$ or $5/x = 40/100$

10. $160:x = 4:3$ or $160/x = 4/3$

Assignment Sheet #5

1. $5a + 3b$

2. $a + 3b$

3. $12.5a$

4. $2a + 1$

5. $a^2b^2c^2 + abc$

6. $47a + 6$

7. $5a^2b^3 + 3ab$

8. $3a + 2$

9. $2a + 2b + a^2 + b^2$

10. $46x^2y$

SOLVE ALGEBRAIC EQUATIONS
UNIT IV

UNIT OBJECTIVE

After completion of this unit, the student should be able to solve algebraic equations. This knowledge will be evidenced by correctly performing the procedures outlined in the assignment sheets and by successfully scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Solve for unknown using a one-step algebraic solution.
2. Solve for unknown using a multi-step algebraic solution.
3. Clear fractions in equations.
4. Solve fractions in equations.

SOLVE ALGEBRAIC EQUATIONS
UNIT IV

SUGGESTED ACTIVITIES

I. Provide student with objective sheet.

II. Provide student with information and assignment sheets.

III. Discuss unit and specific objectives.

IV. Discuss information and assignment sheets.

V. Assist students as they work through assignment sheets.

VI. Give test.

VII. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective sheet

B. Information sheet

C. Assignment sheets-

1. Assignment Sheet #1-- Solve One-Step Algebraic Equation

2. Assignment Sheet #2--Solve Multi-Step Algebraic Equation

3. Assignment Sheet #3--Clear Fractions in Equations

4. Assignment Sheet #4-- Solve Algebraic Word Problems

D. Answers to Assignment Sheets

E. Test

F. Answers to test

II. References

A. Chant, Alfred E., Barbara J. Montalto and Barbara A. Enfinger. Reaching Math Competence New York, NY: Globe Book Company, Inc., Cambridge Division, 1981.

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SOLVE ALGEBRAIC EQUATIONS
UNIT IV

INFORMATION SHEET

I. Terms and Definitions

A. Equation--A mathematical statement of two quantities that are equal

B. Inverse operation--The operation opposite to a given operation

Examples:	Given Operation	Inverse Operation
	addition	subtraction
	subtraction	addition
	multiplication	division
	division	multiplication

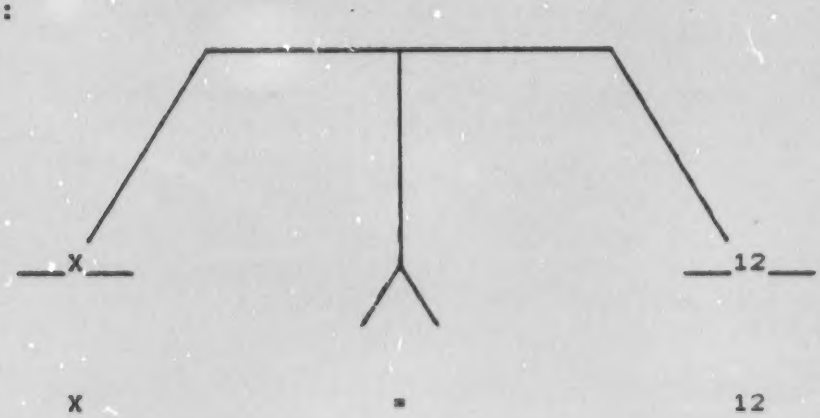
C. Transposition--The removal of a quantity from one side of an equation to the other side by changing the sign of the quantity

Example:
$$x + 5 = 10$$
$$x = 10 - 5$$
$$x = 5$$

II. Solving Equations--Isolate the unknown on one side of the equation

(NOTE: Any change made on one side of an equation must also be made on the other side in order to keep the equation balanced.)

Example:



(NOTE: If we add 2 then x + 2 = 12 + 2;
if we subtract 2 then x -2 = 12 - 2;
if we multiply by 2 then 2x = 2(12);
if we divide by 2 then x/2 = 12/2.)

A. Inverse Operations--Perform the inverse operation to remove a term

Examples:

$$\begin{array}{r} x + 7 = 12 \\ -7 \quad -7 \\ \hline x \quad = 5 \end{array}$$

(NOTE: Isolate x by subtracting 7 from both sides of the equation.)

$$\begin{array}{r} y - 5 = 15 \\ +5 \quad +5 \\ \hline y \quad = 20 \end{array}$$

(NOTE: Isolate y by adding 5 to both sides.)

$$\begin{array}{l} 3a = 36 \\ 3a/3 = 36/3 \\ a = 12 \end{array}$$

(NOTE: Isolate a by dividing both sides by 3.)

$$\begin{array}{l} n/2 = 5 + 3 \\ (n/2)(2) = 2(5+3) \\ n = 10 + 6 \\ n = 16 \end{array}$$

(NOTE: Isolate n by multiplying both side by 2)

(NOTE: When using multiplication as an inverse operation, be sure to multiply all of the terms on both sides, not just one term.)

B. Transposition--A term may be moved from one side of an equation to the other side by changing the sign of the term being moved. This accomplishes the same purpose as inverse operations.

Example:
$$\begin{array}{l} x + 15 = 23 \\ x = 23 - 15 \\ x = 8 \end{array}$$

(NOTE: In the previous example, move the + 15 to the other side and change the sign to - 15.)

(NOTE: The following illustrates a problem solved by inverse operation and by transposition.)

Examples:

Inverse Operation	Transpostition
$x + 2 = 15$	$x + 2 = 15$
$\quad - 2 \quad -2$	$x \quad = 15 - 2$
$x \quad = 13$	$x \quad = 13$

C. Multiple Inverse Operations--If more than one operation is needed, do the addition and subtraction first, then do the muliplication and divisiion.

Example:

$$5a + 8 = 43$$
$$5a = 43 - 8 \quad \text{(Subtraction)}$$
$$5a = 35$$

$$\frac{5a}{5} = \frac{35}{5} \quad \text{(Divisiion)}$$

$$a = 7$$

D. Collect Terms in Equations--Like terms in equations may be combined. A quantity may be moved from one side to the other by changing the sign of the quantity.

Example: $5x + 3 = 18 + 3x$

(NOTE: The task in this example is to combine like terms and isolate the unknown, x , on one side of the equation.)

Step 1: Move the smaller term (variable) to the opposite side of the equation, changing its sign.

$$5x - 3x + 3 = 18$$
$$2x + 3 = 18$$

Step 2: Move the smaller constant to the opposite side, changing its sign.

$$3x = 18 - 3$$
$$3x = 15$$

Step 3: Divide both sides by 3.

$$(3x)/3 = 15/3$$
$$x = 5$$

E. Solving Fractional Equations--When an equation has fractions on both sides, find a common denominator and multiply each term by the common denominator.

Example: $x/2 + 5 = x/3 + 8$

Step 1: Determine the common denominator. In this case, 6 is the common denominator for 2 and 3.

Step 2: Multiply each term by the common denominator.

(NOTE: Put all of the expressions in parentheses.)

$$6(x/2+5) = 6(x/3+8)$$

$$6x/2 + 30 = 6x/3 + 48$$

Step 3: Reduce the improper fractions to their lowest terms.

$$3x + 30 = 2x + 48$$

Step 4: Combine like terms.

$$3x - 2x = 48 - 30$$

$$x = 18$$

F. Checking Solutions to Equations--Substitute the solution for the unknown (variable) and perform the indicated operation. If the equation "balances", the solution is correct.

Example:

Problem	Check
$5a + 8 = 43$	$5(7) + 8 = 43$
$5a = 43 - 8$	$35 + 8 = 43$
$5a = 35$	$43 = 43$
$a = 7$	

G. Solve Word Problems Using Algebra

1. Read the problem carefully and decide exactly what you are being asked to find.

2. Represent the unknown with a specific variable.

3. Write an equation which expresses the relationship between the unknown and the quantities given in the problem.

4. Solve the equation.

5. Make a logic check. Check your answer by substituting the solution in the place of the variable. Check the reasonableness of the answer by determining whether or not it satisfies the conditions stated in the problem.

Example: A nursing class has 7 less than twice as many students this year as last year. There are 43 students in the class this year. Find the number of students in the class last year.

1. You are being asked to find the number of students in class last year.

2. x = number of students in class last year.

3. $2x - 7 = 43$

4. $2x - 7 = 43$
   $2x = 43 + 7$
   $2x = 50$
   $x = 25$

5. Logic check: This year's class (43) is 7 less than twice last year's class, 2 (25) or 50.

$$43 = 2\,(25) - 7$$
$$43 = 50 - 7$$
$$43 = 43$$

SOLVE ALGEBRAIC EQUATIONS
UNIT IV

ASSIGNMENT SHEET #1--SOLVE ONE-STEP ALGEBRAIC EQUATIONS

Directions: Solve the following equations using either inverse operations or transposition.

1. $6x = 12$

2. $x/2 = 36$

3. $x + 16 = 22$

4. $x - 16 = 22$

5. $x + 5 = 15$

6. $x - 12 = 111$

7. $5x = 125$

8. $x/8 = 12$

9. $x/36 = 12$

10. $x + 76 = 118$

SOLVE ALGEBRAIC EQUATIONS
UNIT IV

ASSIGNMENT SHEET #2--SOLVE MULTI-STEP ALGEBRAIC EQUATIONS

Directions: Solve for x in the following equations.

1. $3x - 5 = 16$

2. $8x - 7 = 65$

3. $x/5 + 4 = 54$

4. $15x - 3 = 42$

5. $3x + 19 = 46$

6. $5x/2 + 3 = 13$

7. $15x + 5 = 95$

8. $60x - 7 = 173$

9. $26x - 15 = 37$

10. $35x + 12 = 82$

SOLVE ALGEBRAIC EQUATIONS
UNIT IV

ASSIGNMENT SHEET #3--CLEAR FRACTIONS IN EQUATIONS

Directions: Solve for x in the following equations.

1. $\frac{x}{2} - 3 = \frac{x}{7} + 2$

2. $\frac{x}{5} - 7 = \frac{x}{10} + 8$

3. $\frac{x}{4} - 8 = \frac{x}{8} - 2$

4. $\frac{x}{7} + 8 = x + 2$

5. $x + 5 = \frac{2x}{7} + 10$

6. $\frac{x}{3} - 6 = \frac{x}{9}$

7. $\frac{x}{3} + 2 = \frac{x}{9} + 6$

8. $\frac{x}{2} - 5 = \frac{x}{14} + 1$

9. $\frac{x}{4} + 5 = \frac{x}{8} + 6$

10. $\frac{x}{3} + 5 = \frac{x}{6} + 6$

SOLVE ALGEBRAIC EQUATIONS
UNIT IV

ASSIGNMENT SHEET #4--SOLVE ALGEBRAIC WORD PROBLEMS

Directions: Set up equations and solve the following problems.

1. A carpenter received a $.20 an hour raise this year. Last year the carpenter earned $375.20 for a 40-hour work week. What is the carpenter's hourly wage this year?

2. A car salesman sold 2 1/2 times more cars in October than in September. The salesman sold 30 cars in October. How many cars did he sell in September?

3. A 12 ft. piece of pipe was cut into two unequal lengths. One piece is three times as long as the other. What is the length of the longer piece? What is the length of the shorter piece?

4. A builder estimates that labor costs for a job will be $35 more than material costs. Estimated total costs for the materials and labor is $105. What will be the estimated labor cost? What will be the estimated material costs?

5. Four holes are drilled along a straight line in a plate. The distance between Hole 1 and Hole 4 is 35 centimeters. The distance between Hole 2 and Hole 3 is twice the distance betwen Hole 1 and Hole 2. The distance between Hole 3 and Hole 4 is the same as the distance between Hole 2 and Hole 3. What is the distance between Hole 1 and Hole 3?

SOLVE ALGEBRAIC WORD PROBLEMS
UNIT IV

ANSWERS TO ASSIGNMENT SHEETS

Assignment Sheet #1

1. 2
2. 72
3. 6
4. 38
5. 10
6. 123
7. 25
8. 256
9. 432
10. 42

Assignment Sheet #2

1. 7
2. 9
3. 250
4. 3
5. 9
6. 4
7. 6
8. 30
9. 2
10. 2

Assignment Sheet #3

1. 14
2. 10
3. 48
4. 7
5. 7
6. 27
7. 18
8. 14
9. 8
10. 6

Assignment Sheet #4

1. $9.58
2. 12
3. shorter = 3; longer = 9
4. material = $35; labor = $35
5. 21 cm

SOLVE ALGEBRAIC EQUATIONS
UNIT IV

TEST

Directions: Solve for the unknown in the following problems:

1. a. $3x = 21$

   b. $x + 52 = 94$

   c. $x - 56 = 31$

   d. $\frac{x}{5} = 125$

   e. $7x = 105$

2. a. $\frac{x}{3} = 5$

   b. $2x - 5 = 7$

   c. $5x - 7 = 2x + 8$

   d. $\frac{2a}{8} = 5$

   e. $x + 5 = 7$

3. a. $\frac{3a}{2} + 2 = \frac{a}{4} + 7$

   b. $\frac{a}{4} + 7 = \frac{a}{8} + 9$

   c. $\frac{a}{2} - 4 = \frac{a}{9} + 3$

d. $\frac{x}{3} - 1 = \frac{x}{6} + 1$

e. $x + 5 = \frac{2x}{7} + 10$

4. a. Jane is twice as old as her sister. The sum of their ages is 42 years. How old is each girl?

b. From Little Rock to Ft. Smith is twice as far as from Little Rock to Memphis. The two distances total 240 miles. What is the mileage in each case?

c. John and Tim have $7.20. Tim has 5 times as much as John. How much does each man have?

d. A skilled worker earns three times as much as an unskilled worker. Together they earn $14.60 per hour. What is the hourly wage for each?

e. A machinist receives as take-home pay three-fourths of his salary after deductions are taken out. His take-home pay is $586.50 per week. What is his weekly salary before deductions?

SOLVE ALGEBRAIC WORD PROBLEMS
UNIT IV

ANSWERS TO TEST

1. a. x = 3
   b. x = 42
   c. x = 31
   d. x = 125
   e. x = 105

2. a. x = 15
   b. x = 6
   c. x = 5
   d. x = 20
   e. x = 2

3. a. a = 4
   b. a = 18
   c. a = 16
   d. x = 6
   e. x = 7

4. a. sister = 14; Jane = 28
   b. Little Rock Ft. Smith = 160; Little Rock-Memphis = 80
   c. Tim = $6.00; John = $1.20
   d. unskilled = $3.65; skilled = $10.95
   e. $782.

BINOMIALS/POLYNOMIALS
UNIT V

UNIT OBJECTIVE

After completion of this unit, the student should be able to add, subtract, multiply, and divide binomial and polynomial numbers. This knowledge will be evidenced by correctly performing the procedures outlined in the assignment sheets and by scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Add binomials/polynomials.
2. Subtract binomials/polynomials.
3. Multiply binomials/polynomials.
4. Divide binomials/polynomials.

BINOMIALS/POLYNOMIALS
UNIT V

SUGGESTED ACTIVITIES

I. Provide student with objective sheet.

II. Provide student with information and assignment sheets.

III. Discuss unit and specific objectives.

IV. Discuss information and assignment sheets.

V. Assist students as they work through assignment sheets.

VI. Give test.

VII. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective sheet

B. Information sheet

C. Assignment sheets-

1. Assignment Sheet #1--Adding binomials/polynomials

2. Assignment Sheet #2--Subtracting binomials/polynomials

3. Assignment Sheet #3--Multiplying binomials/polynomials

4. Assignment Sheet #4--Dividing binomials/polynomials

D. Answers to Assignment Sheets

E. Test

F. Answers to test

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BINOMIALS/POLYNOMIALS
UNIT V

INFORMATION SHEET

I. Terms and Definitons

A. Monomial--An expression of a single quantity

Examples: a, a 4a, 1/2 b, b/2, bc, cf

B. Binomial--Expression made up of two terms; bi- means 2; nomials means terms

Examples: a + b, 2b + 3c

C. Polynomial--Expression made up of more than two terms; poly- means many; nomials means terms

D. Exponent--The number to the right and slightly above the symbol; also called power

Example: a^4

II. Procedures:

A. Adding binomials/polynomials with the same sign--Add the numbers and give the answer the common sign

Examples:

a + 2b	a - c	- a + c	a + b - c
a + 2b	a - c	- a + c	a + b - c
2a + 4b	2a - 2c	-2a + 2c	2a + 2b - 2c

B. Adding binomials/polynomials with different signs--Add the like expressions separately

Examples:

a - b	d + m	a - b - c	- a + b
a + b	- d + m	- a - b + c	2a - 3b
2a	2m	-2b	a - 2b

C. Subtracting binomials/polynomials--Change the sign of the subtrahend and then add

Examples:

$$\begin{array}{r} x + y \\ x - y \\ \hline \end{array} \qquad \text{(changing the bottom sign becomes)} \qquad \begin{array}{r} x + y \\ -x + y \\ \hline 2y \end{array}$$

$$\begin{array}{r} 5x - 7y \\ x + y \\ \hline 4x - 8y \end{array} \qquad \begin{array}{r} 5b + 3c \\ -b + c \\ \hline 6b + 2c \end{array}$$

D. Multiplying binomials/polynomials--Follow these three steps using this example

Example:
$$\begin{array}{r} a + b \\ a + b \\ \hline \end{array}$$

Step 1. Multiply the bottom left term times the top terms, moving left to right

$$\begin{array}{r} a + b \\ a \\ \hline a^2 + ab \end{array}$$

Step 2. Multiply the next bottom term times the top terms, again moving left to right

$$\begin{array}{r} a + b \\ b \\ \hline ab + b^2 \end{array}$$

Step 3. Line up the results of Step 1 and Step 2 so that like terms are in columns and combine the results

$$\begin{array}{r} a^2 + ab \\ ab + b^2 \\ \hline a^2 + 2ab + b^2 \end{array}$$

(NOTE: Remember to add the exponents.)

E. Dividing binomials/polynomials--Follow these three steps using this example

Example:

$$a + b \overline{)\ a^2 + 2ab + b^2}$$

Step 1. Divide the first divisor letter, (in this case, a) into the first dividend letter, (in this case, a^2.)

(NOTE: Be sure to multiply partial answer times the divisor and subtract the product as in usual division.)

$$\begin{array}{r} a \\ a + b \overline{)\ a^2 + 2ab + b^2} \\ a^2 + ab \\ \hline ab + b^2 \end{array}$$

Step 2. Put the sign between the second and third expressions (in this case, the + between 2ab and ^{2}) in the answer.

$$\begin{array}{r} a + \\ a + b \overline{)\ a^2 + 2ab + b^2} \\ a^2 + ab \\ \hline ab + b^2 \end{array}$$

Step 3. Repeat the above steps, using the last letter in the divisor (b, in this case) and the last expression in the dividend, (b^2, in this case.)

$$\begin{array}{r} a + b \\ a + b \overline{)\ a^2 + 2ab + b^2} \\ a^2 + ab \\ \hline ab + b^2 \\ ab + b^2 \\ \hline \end{array}$$

Examples:

$$
\begin{array}{r}
x \quad + 2 \\
\hline
x + 1 \;\big)\; x^2 + 3x + 2 \\
\underline{x^2 + x} \\
2x + 2 \\
\underline{2x + 2} \\
0
\end{array}
$$

$$
\begin{array}{r}
2x \quad + 3y \\
\hline
ax + 3y \;\big)\; 4x^2 + 12xy + 9y^2 \\
\underline{4x^2 + 6xy} \\
6xy + 9y^2 \\
\underline{6xy + 9y^2} \\
0
\end{array}
$$

BINOMIALS/POLYNOMIALS
UNIT V

ASSIGNMENT SHEET #1--ADDING BINOMIALS/POLYNOMIALS

Directions: Add the following problems. Be sure to apply the rules for adding signed numbers.

a. $5a + 3b$
$2a + 3b$

b. $6x + 7y$
$2x + 11y$

c. $4x - 3b$
$5x - 2b$

d. $9c + 3d$
$4c + 5d$

e. $5a + 3c$
$-9a + 4c$

f. $2x + 5y$
$-3x - 2y$

g. $-12m + 2n$
$4m - 6n$

h. $6b - 3c$
$2b + 5c$

i. $9c - 2d$
$4c + 2d$

j. $2x + y$
$3x - y$

k. $5a - b$
$a + b$

l. $7b + 3c$
$4b - 3c$

m. $5x + 3y$
$2x + 4y$

n. $6x + 2y$
$2x + 6y$

o. $6x + 2y$
$2x - 2y$

p. $5m + 3n$
$-2m - 5n$

BINOMIALS/POLYNOMIALS
UNIT V

ASSIGNMENT SHEET #1--ADDING BINOMIALS/POLYNOMIALS

Directions: Add the following problems. Be sure to apply the rules for adding signed numbers.

a. $5a + 3b$
$2a + 3b$

b. $6x + 7y$
$2x + 11y$

c. $4x - 3b$
$5x - 2b$

d. $9c + 3d$
$4c + 5d$

e. $5a + 3c$
$-9a + 4c$

f. $2x + 5y$
$-3x - 2y$

g. $-12m + 2n$
$4m - 6n$

h. $6b - 3c$
$2b + 5c$

i. $9c - 2d$
$4c + 2d$

j. $2x + y$
$3x - y$

k. $5a - b$
$a + b$

l. $7b + 3c$
$4b - 3c$

m. $5x + 3y$
$2x + 4y$

n. $6x + 2y$
$2x + 6y$

o. $6x + 2y$
$2x - 2y$

p. $5m + 3n$
$-2m - 5n$

BINOMIALS/POLYNOMIALS
UNIT V

ASSIGNMENT SHEET #2--SUBTRACTING BINOMIALS/POLYNOMIALS

Directions: Subtract the following binomials. Remember the rule for subraction: Change the signs of the subtrahend, then add.

a. $\begin{array}{r} a + b \\ a - b \\ \hline \end{array}$

b. $\begin{array}{r} b + c \\ b - c \\ \hline \end{array}$

c. $\begin{array}{r} b - c \\ -b - c \\ \hline \end{array}$

d. $\begin{array}{r} -b + c \\ -2b + c \\ \hline \end{array}$

e. $\begin{array}{r} 6x - 2c \\ -4x + 5c \\ \hline \end{array}$

f. $\begin{array}{r} 3x - 2c \\ 3x - 2c \\ \hline \end{array}$

g. $\begin{array}{r} 2c - d \\ -2c + d \\ \hline \end{array}$

h. $\begin{array}{r} - x + y \\ - x - y \\ \hline \end{array}$

i. $\begin{array}{r} 5x - 7y \\ x + y \\ \hline \end{array}$

j. $\begin{array}{r} 3x + 5y \\ 2x - y \\ \hline \end{array}$

k. $\begin{array}{r} 8a - b \\ 3a + 2b \\ \hline \end{array}$

l. $\begin{array}{r} 3b - 2c \\ 2b + 2c \\ \hline \end{array}$

m. $\begin{array}{r} -2x - 12y \\ -5x + 4y \\ \hline \end{array}$

n. $\begin{array}{r} 7a + 3b \\ -4a - 3b \\ \hline \end{array}$

o. $\begin{array}{r} 5b + 3c \\ - b - c \\ \hline \end{array}$

p. $\begin{array}{r} -8m + 6n \\ -5m + 4n \\ \hline \end{array}$

BINOMIALS/POLYNOMIALS
UNIT V

ASSIGNMENT SHEET #3--MULTIPLYING BINOMIALS/POLYNOMIALS

Directions: Multiply the following binomials. Remember to add the exponents and start from the left side of the bottom number.

a. $\begin{array}{r} a + b \\ a + b \\ \hline \end{array}$

b. $\begin{array}{r} 3x - 2y \\ 2x - 3y \\ \hline \end{array}$

c. $\begin{array}{r} 2c + d \\ c + 2d \\ \hline \end{array}$

d. $\begin{array}{r} 3x - 2y \\ 2x - y \\ \hline \end{array}$

e. $\begin{array}{r} 5a + 2b \\ 2a + 3b \\ \hline \end{array}$

f. $\begin{array}{r} 4a - 2c \\ a - c \\ \hline \end{array}$

g. $\begin{array}{r} 3x + 2y \\ 2x - 3y \\ \hline \end{array}$

h. $\begin{array}{r} 2x + 4y \\ 3x - 2y \\ \hline \end{array}$

i. $\begin{array}{r} 4a - 2c \\ a - c \\ \hline \end{array}$

j. $\begin{array}{r} x - 5 \\ x + 5 \\ \hline \end{array}$

k. $\begin{array}{r} 2x - 3 \\ 2x + 3 \\ \hline \end{array}$

l. $\begin{array}{r} 3a - 4b \\ a + 3b \\ \hline \end{array}$

BINOMIALS/POLYNOMIALS
UNIT V

ASSIGNMENT SHEET #4--DIVIDING BINOMIALS/POLYNOMIALS

Directions: Divide the following problems. Remember to subtract exponents.

a. $a + 2 \overline{)a^2 + 4a + 4}$

b. $a - 2 \overline{)a^2 - 4a + 4}$

c. $x - 5 \overline{)x^2 - 6x + 5}$

d. $x + 3 \overline{)x^2 + x - 6}$

e. $x + 7 \overline{)x^2 + 4x - 21}$

f. $x - 5 \overline{)x^2 + 3x - 40}$

g. $2x+3y \overline{)2x^2 + 7xy + 6y^2}$

h. $3x-2y \overline{)6x^2+5xy-6y^2}$

i. $a - b \overline{)2a^2 - ab - b^2}$

j. $a + 4 \overline{)a^2 + 2a - 8}$

k. $c - 3 \overline{)c^2 - c - 6}$

l. $3 - 2x \overline{)6 + 5x - 6x^2}$

BINOMIALS/POLYNOMIALS
UNIT V

ANSWERS TO ASSIGNMENT SHEETS

Assignment Sheet #1

a. $7a + 6b$	b. $8x + 18y$	c. $9x - 5b$	d. $13c + 8d$
e. $-4a + 7c$	f. $-x + 3y$	g. $-8m - 4n$	h. $8b + 2c$
i. $13c$	j. $5x$	k. $6a$	l. $11b$
m. $7x + 7y$	n. $8x + 8y$	o. $8x$	p. $3m - 2n$

Assignment Sheet #2

a. $2b$	b. $2c$	c. $2b$	d. b
e. $10x - 7c$	f. 0	g. $4c - 2d$	h. $2y$
i. $4x - 8y$	j. $x + 6y$	k. $5a - 3b$	l. $b - 4c$
m. $3x - 16y$	n. $11a + 6b$	o. $6b + 4c$	p. $-3m + 2n$

Assignment Sheet #3

a. $a^2+2ab+b^2$	b. $6x^2-13xy+6y^2$
c. $2d^2+5cd+2d^2$	d. $6x^2-7xy+2y^2$
e. $10a^2+19ab+6b^2$	f. $4a^2-6ac+2c^2$
g. $6x^2-5xy-6y^2$	h. $6x^2+8xy-8y^2$
i. $4a^2-6ac+2c^2$	j. x^2-25
k. $4x^2-9$	l. $3a^2+5ab-12b^2$

Assignment Sheet #4

a. $a + 2$	b. $a + 2$
c. $x - 1$	d. $x - 2$
e. $x - 3$	f. $x + 8$
g. $x + 2y$	h. $2x + 3y$
i. $2a + b$	j. $a - 2$
k. $c + 2$	l. $2 + 3x$

BINOMIALS/POLYNOMIALS
UNIT V

TEST

Directions: Solve the following problems. Perform the operation indicated.

1. Add:

a. $13a + 2c$ / $5a - 2c$

b. $13a - 2c$ / $-5a - 2c$

c. $4a - 2b$ / $-3a - 2b$

d. $2a^2-2c^2$ / $2a^2-5c^2$

2. Subtract:

a. $4a - 2c$ / $-2a + c$

b. $3a^2 + 2ab$ / $-a^2 - ab$

c. $7x - y$ / $6x - 2y$

d. $7a + 3b$ / $-4a - 3b$

3. Multiply:

a. $x - y$ / $3x + 2y$

b. $5a - 3b$ / $-a - 3b$

c. $x + 8$ / $x + 8$

d. $x - 5$ / $x - 5$

e. $3x + 4$ / $3x - 4$

f. $3x + 5$ / $x + 1$

g. $8m + 2n$ / $2m - 5n$

4. Divide:

a. $x+3\overline{)x^2+6x+9}$

b. $x-3\overline{)x^2-13x+30}$

c. $2x+4\overline{)2x^2+10x+12}$

d. $x+7\overline{)x^2+4x-21}$

e. $5+x\overline{)25+10x+x^2}$

f. $3x-2y\overline{)6x^2+5xy-6y^2}$

BINOMIALS/POLYNOMIALS
UNIT V

ANSWERS TO TEST

1. a. $18a$ b. $8a - 4c$ c. $a - 4b$ d. $4a^2-7c^2$

2. a. $6a-3c$ b. $4a^2+3ab$ c. $x + y$ d. $11a +6b$

3. a. $3x^2-xy-2y^2$ b. $-5a^2-12ab+9b^2$
   c. $x^2+16x+64$ d. $x^2-10x+25$
   e. $9x^2-16$ f. $3x^2+8x+5$
   g. $16m^2-36n-10n^2$

4. a. $x + 3$ b. $x + 10$
   c. $x + 3$ d. $x - 3$
   e. $5 + x$ f. $2x + 3y$

SIMULTANEOUS EQUATIONS
UNIT VI

UNIT OBJECTIVE

After completion of this unit, the student should be able to solve simultaneous equations. This knowledge will be evidenced by correctly performing the procedures outlined in the assignment sheets and by scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Simplify and factor algebraic expressions as follows:
   a. factor into prime factor containing common monomial factors
   b. factor difference of two squares
   c. factor trinomial expressions
2. Manipulate integral and fractional exponents:
   a. manipulate integral and fractional exponents
   b. express radical in simplest form
   c. perform basic operations with radicals
3. Solve simultaneous equations:
   a. factor given algebraic expressions
   b. solve simultaneous equations word problems

SIMULTANEOUS EQUATIONS
UNIT VI

SUGGESTED ACTIVITIES

I. Provide student with objective sheet.

II. Provide student with information and assignment sheets.

III. Discuss unit and specific objectives.

IV. Discuss information and assignment sheets.

V. Assist students as they work through assignment sheets.

VI. Give test.

VII. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective sheet

B. Information sheet

C. Assignment sheets-

1. Assignment Sheet #1--Factor common monomial factors

2. Assignment Sheet #2--Factor difference of two squares

3. Assignment Sheet #3--Factor trinomial expressions

4. Assignment Sheet #4--Manipulate integral exponents

5. Assignment Sheet #5--Manipulate fractional exponents

6. Assignment Sheet #6--Express radicals in simplest forms

7. Assignment Sheet #7--Perform basic operations with radicals

8. Assignment Sheet #8--Factor given algebraic

expressions

9. Assignment Sheet #9--Solve simultaneous equations

10. Assignment Sheet #10--Solve word problems

D. Answers to Assignment Sheets

E. Test

F. Answers to test

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SIMULTANEOUS EQUATIONS
UNIT VI

INFORMATION SHEET

I. Terms and Definitions

A. Simultaneous--At the same time; the same values of the literal factors exist in both equations

B. Prime factor--A factor is said to be prime when it can be divided into no number other than 1 or itself

C. Product--Multiplications

D. Variables--Having no fixed quantitative value; a quantity capable of assuming any of a set of values

E. Manipulate--To handle, control, or move skillfully

F. Integral--A complete thing

G. Exponents--The number to the right and slightly above the symbol (the 2 in 5^2) is called the exponent

H. Radicals--Not in simplest form; $\sqrt{}$, symbol used to indicate a root of a number

II. Procedures

A. Simplify and Factor Algebraic Expressions

1. Factor into prime factors algebraic expressions containing common monomial factors

Example:

$2x^4 + 3x^3 + x^2$	(X^2 is common in all 3)
$x^2(2x^2) + x^2(3x) + x^2(1)$	(remove x^2 from all)
$x^2(2x^2 + 3x + 1)$	(common factor x^2 as coefficient)

Example:

$15a^2b = 3 \cdot 5 \cdot a \cdot a \cdot b$
Factors of 15 = $3 \cdot 5$
Factors of a^2 = $a \cdot a$
Factors of b = $b \cdot 1$

2. Difference of Two Squares--A binomial expression such as $x^2 - 16$ in which each term is a perfect square and the terms are separated by a minus (-) sign is called a "difference of two squares"

   Example:

   $x^2 - 16 = (x-4)(x+4)$

   (NOTE: Factor of x^2 is x; square factor of 16 = 4; one factor has negative sign (-), the other has a positive (+) sign.)

3. Factor Trinomial Expressions

   a. A trinomial should be inspected first for a common factor

      Example:

      $3x^2 + 15x + 18$ (3 is common to all three)
      $3(x^2 + 5x + 6) = 3(x+3)(x+2)$

   b. If the sign of the third term in the trinomials is positive (+), the signs between the binomial factors will be the same, such as both positive (+) or both negative (-)

      (NOTE: Remember that negative times negative equals positive.)

   c. If third term is negative, then the binomial factors will have unlike signs

      Example:

      $x^2 + 3x - 18 = (x+6)(x-3)$

      Proof:
      $$\begin{array}{r} x+6 \\ \underline{x-3} \\ x^2+6x \\ \underline{-3x-18} \\ x^2+3x-18 \end{array}$$

B. Manipulate Integral and Fractional Exponents

   1. Manipulate Integral Exponents--Laws of Exponents

a. In multiplication, add the exponents

Examples:

$a^2 \cdot a^3 = a^{2+3} = a^5$

$b^3 \cdot b \cdot b^4 = b^{3+1+4} = b^8$

b. In division, subtract the exponents

Example:

$$\frac{a^4}{a^2} = a^{4-2} = a^2$$

c. When a power is raised to a power, exponents are multiplied

Example:

$(x^2)3 = x^{2.3} = x^6$

2. Manipulate Fractional Exponents--The above laws have been expressed as positive whole numbers; the following explains fractions

(NOTE: The term $a^{1/2}$ has the same meaning as $\sqrt{a}$.)

(NOTE: For any base with a fractional exponent, the numerator of the exponent denotes the power to which the base is to be raised and the denominator of exponent denotes the root to be extracted.)

Example:

$27^{2/3} = \sqrt[3]{27^2} = \sqrt[3]{729} = 9$

3. Express Radicals in Simplest Form

a. The root of a product equals the product of the roots of its factors

Examples:

$\sqrt{36} = \sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6$

$$\sqrt{a^2b^2c} = \sqrt{a^2} \cdot \sqrt{b^2} \cdot \sqrt{c} = ab\sqrt{c}$$

b. In the case of fractions, simplify so that no radicals appear in the denominator:

Example:

$$\sqrt{\frac{x}{n}} = \frac{\sqrt{x}}{\sqrt{n}} = \frac{\sqrt{x \cdot n}}{\sqrt{n \cdot n}} = \frac{\sqrt{xn}}{\sqrt{n^2}} = \frac{\sqrt{xn}}{n}$$

4. Perform Basic Operations with Radicals

a. Addition and subtractions will be like other algebraic terms; "like" radicals may be added or subtracted

Example:

$\sqrt{3} + 4\sqrt{5} - 3\sqrt{3} + \sqrt{5} =$

(NOTE: $\sqrt{3}$ and $\sqrt{5}$ are alike)

$(\sqrt{3} - 3\sqrt{3}) + (4\sqrt{5} + \sqrt{5})$ = (Group like terms)

$-2\sqrt{3} + 5\sqrt{5}$ (answer)

b. Multiplication of Radicals

(NOTE: This process is the same as any algebraic multiplication.)

Example:

$\sqrt{2}(\sqrt{6} + 3\sqrt{5})$ (Expression)

$\sqrt{2} \cdot \sqrt{6} + \sqrt{2 \cdot 3}\sqrt{5}$ ($\sqrt{2}$ by everything)

$\sqrt{12} + 3\sqrt{10}$ =

$\sqrt{4 \cdot 3} + 3\sqrt{10}$ (No perfect factor)

$2\sqrt{3} + 3\sqrt{10}$

c. Division of radicals--Rationalize (simplify) by multiplying both the numerator and the denominator by a "surd", which is a binomial with like terms as the binomial in the denominator but with the sign between terms changed.

Example:

$$\frac{\sqrt{3}}{\sqrt{2}-\sqrt{5}} = \frac{\sqrt{3}(\sqrt{2}+\sqrt{5})}{(\sqrt{2}-\sqrt{5})(\sqrt{2}+\sqrt{5})} \overset{\text{surd}}{=} \frac{\sqrt{6}+\sqrt{15}}{2-5} = -\frac{\sqrt{6}+\sqrt{15}}{3}$$

(NOTE: In the above example, to keep the denominator positive, move the sign across the equals sign and make it negative in the last step.)

d. Solve a radical equation--arrange the equation so that the radical with the unknown is the only term on the left side of the equation

Example: Solve for x

$$\sqrt{3x} = 6$$
$$(\sqrt{3x})^2 = 6^2$$
$$3x = 36$$
$$x = 12$$

C. Solve Simultaneous Equations

1. Factor given algebraic expressions

(NOTE: Think of it as the reverse of multiplication.)

a. Extract the square root of the first term

b. Extract the square root of the last term

c. Check to see if twice the product of these two square roots results in the middle term

Example:

$a^2 + 2ay + y^2$ Expression
$a^2 = a$ (square root of a^2) First step
$y^2 = y$ (square root of y^2) Second step
$a \cdot y \cdot 2 = 2ay$ Third step
Therefore, factors will be $(a+y)(a+y)$

2. Solve Simultaneous Equations--The same values of the literal factors exist in both equations

Example:

$$\begin{array}{r} a+b = 15 \\ a-b = \ \ 5 \\ \hline 2a \ \ = 20 \\ a \ \ = 10 \end{array}$$

Follow the rules for adding and subtracting

$$\begin{array}{rl} a + b = 15 & \text{Substitute 10 for a to solve for b} \\ 10 + b = 15 & \\ b = 15 - 10 & \\ b = 5 & \end{array}$$

(NOTE: When the coefficients are different, the equation may be changed if it remains equal on both sides of the =)

Example:

$$\begin{array}{rl} 2x + 3y = 9 & \\ x - 2y = 1 & \text{Multiply this bottom equation by 2 to make one of the coefficients equal to the top one.} \end{array}$$

Rewritten:

$$\begin{array}{r} 2x + 3y = 9 \\ 2x - 2y = 2 \\ \hline y = 11 \end{array}$$

Now solve for x in the original problem

$$\begin{array}{l} 2x + 3y = 9 \\ 2x + 3(11) = 9 \\ 2x + 33 = 9 \\ 2x = 9-33 \\ 2x = -24 \\ x = -12 \end{array}$$

SIMULTANEOUS EQUATIONS
UNIT VI

ASSIGNMENT SHEET #1--FACTOR COMMON MONOMIAL FACTORS

Directions: Find common monomial factors.

1. $2x^3 + 3x^2 + x =$

2. $9x^2y^3 + 6yx^4 + 3y^5 =$

3. $b^4 + b^3 + b^2 =$

4. $b^4 - b^3 - b^2 =$

5. $x^4y^2 + x^2y - 2xy^2 =$

6. $a^3 + a^2 + a^2 =$

7. $a^4n^3 + a^2n^2 + a^2n =$

8. $6x^2 + 4x =$

9. $5b^4 + 10b^2 + 15b =$

10. $x^4b^3c^2 + x^2bc =$

11. $15a^2b =$

12. $36a^4b^2 =$

13. $54x^3b^3 =$

14. $19a^2b^2 =$

SIMULTANEOUS EQUATIONS
UNIT VI

ASSIGNMENT SHEET #2--FACTOR THE DIFFERENCE OF TWO SQUARES

Directions: Factor the following terms.

1. $x^2 - 4 =$

2. $b^4 - 16 =$

3. $9x^2 - 25 =$

4. $16 - 25c^2 =$

5. $25b^6 - 36 =$

6. $b^2 - c^2 =$

7. $4x^4 - 9b^4 =$

8. $16x^2 - 100 =$

9. $64x^4 - 81 =$

10. $49m^2 - 25 =$

11. $144 - x^2 =$

12. $m^2 - 169 =$

13. $b^2 - 49 =$

14. $25x^2 - 36c^2 =$

15. $L^2 - 4 =$

16. $d^4 - 16 =$

17. $x^2 - 121 =$

18. $x^4 - 81 =$

19. $100 - 16x^2 =$

20. $81 - 81x^2 =$

SIMULTANEOUS EQUATIONS
UNIT VI

ASSIGNMENT SHEET #3--FACTOR SIMPLE TRINOMIAL EXPRESSIONS

Directions: Factor the following trinomials.

1. $x^2 + 5x + 6 =$

2. $x^2 - 5x + 6 =$

3. $x^2 + x - 6 =$

4. $x^2 - x - 6 =$

5. $x^2 - 6x + 8 =$

6. $x^2 + 7x + 12 =$

7. $x^2 + x - 6 =$

8. $x^2 - x + 6 =$

9. $x^2 + 2x - 8 =$

10. $16x^2 - 16x + 3 =$

11. $3x^2 - x - 2 =$

12. $3x^2 + 14x - 5 =$

13. $5x^2 - 24x - 5 =$

14. $6x^2 - 7x - 5 =$

15. $x^2 - 8x + 7 =$

16. $x^2 - 12 - 7x =$

SIMULTANEOUS EQUATIONS
UNIT VI

ASSIGNMENT SHEET #4--MANIPULATE INTEGRAL EXPONENTS

Directions: Simplify the following expressions.

1. $a \cdot a \cdot a =$

2. $a^2 \cdot a^3 \cdot a^4 =$

3. $a^m \cdot a^n =$

4. $a^5 \cdot a^4 =$

5. $x^3 \cdot x^2 =$

6. $x \cdot x^4 =$

7. $m^3 \cdot m^2 =$

8. $x^m \cdot x^a =$

9. $x^4 \cdot x^4 =$

10. $a \cdot a \cdot a \cdot a \cdot a =$

11. $\frac{x^4}{x^2} =$

12. $\frac{m^5}{m^4} =$

13. $\frac{b^3}{b^3} =$

14. $\frac{x^5}{x} =$

15. $\frac{b^{12}}{b^8} =$

16. $\frac{c^5}{c^3}$ =

17. $\frac{m^2}{m^4}$ =

18. $\frac{x^5}{x^{10}}$ =

19. $\frac{a^{15}}{a^2}$ =

20. $\frac{L^5}{L^4}$ =

21. $(a^4)5$ =

22. $(a^3)2$ =

23. $(x^m)n$ =

24. $(x^3)3$ =

25. $(x^2)5$ =

26. $(m^{no})no$ =

27. $(b^4)4$ =

28. $(n^a)a$ =

29. $(c^6)n$ =

30. $(L^n)n$ =

31. $(x^3)5$ =

32. $(x^5)2$ =

SIMULTANEOUS EQUATIONS
UNIT VI

ASSIGNMENT SHEET #5--MANIPULATE FRACTIONAL EXPONENTS

Directions: Take fractional exponents out of the following and express in equivalent radical form.

1. $a^{1/2}$ =

2. $27^{2/3}$ =

3. $(-8)^{2/3}$ =

4. $36^{1/2}$ =

5. $9^{1/2}$ =

6. $a^{1/3}$ =

7. $(a^{1/2})^2$ =

8. $(a^{3/4})4$ =

9. $(x^{2/3})3$ =

10. $(x^{2/3})6$ =

11. $b^{1/3}$ =

12. $w^{1/2}$ =

13. $z^{2/3}$ =

14. $a^{4/3}$ =

15. $x^{-1/2}$ =

16. $p^{2/5}$ =

17. $r^{-2/3}$ =

18. $z^{6/5}$ =

19. $y^{2/3}$ =

20. $2w^{3/4}$ =

SIMULTANEOUS EQUATIONS
UNIT VI

ASSIGNMENT SHEET #6--EXPRESS RADICALS IN SIMPLEST FORM

Directions: Simplify the following.

1. $\sqrt{9}$

2. $\sqrt{16}$

3. $\sqrt[3]{8}$

4. $\sqrt[3]{27}$

5. $\sqrt{27x^4}$

6. $\sqrt{99a^3d}$

7. $5\sqrt{96x^2y}$

8. $\sqrt{12x^2y^4}$

9. $\sqrt{1/3}$

10. $\sqrt{a/b}$

11. $\sqrt{x^3/y}$

12. $\frac{8}{\sqrt{2}}$

13. $\frac{1}{\sqrt{x}}$

14. $\sqrt{9/16x}$

15. $x\sqrt{y/x}$

16. $\sqrt{y^2/y}$

17. $\sqrt{8x^2y}$

18. $\sqrt{16a^2b^4c^2}$

SIMULTANEOUS EQUATIONS
UNIT VI

ASSIGNMENT SHEET #7--PERFORM BASIC OPERATIONS WITH RADICALS

Directions: Solve the following as indicated.

A. Add or subtract as indicated:

1. $5\sqrt{3} - 2\sqrt{3} =$

2. $5\sqrt{5} - \sqrt{80} =$

3. $2\sqrt{1/3} + \sqrt{1/3} =$

4. $7\sqrt{5} - 2\sqrt{5} + 5\sqrt{5} =$

5. $6\sqrt{27} + 5\sqrt{48} =$

6. $\sqrt{x/8} - \sqrt{x/32} =$

B. Multiply and simplify

1. $\sqrt{3} \cdot \sqrt{3} =$

2. $2\sqrt{10} \cdot \sqrt{2} =$

3. $\sqrt{6} \cdot \sqrt{24} =$

4. $8\sqrt{5} \cdot 4\sqrt{15} =$

5. $4\sqrt{5} \cdot 2\sqrt{5} =$

6. $(\sqrt{x - y})2 =$

7. $\sqrt{6Y} \cdot \sqrt{12X^2Y} =$

8. $\sqrt[3]{2} \cdot \sqrt[3]{4}$

C. Divide:

1. $\dfrac{2\sqrt{10}}{\sqrt{8}} =$

2. $\dfrac{3}{3 - \sqrt{2}} =$

3. $\dfrac{8}{3 + \sqrt{7}} =$

4. $\dfrac{9}{3 - 3\sqrt{3}} =$

5. $\dfrac{x + \sqrt{x}}{x - \sqrt{x}} =$

6. $\dfrac{\sqrt{2} + 3}{\sqrt{3} + 2} =$

SIMULTANEOUS EQUATIONS
UNIT VI

ASSIGNMENT SHEET #8--FACTOR GIVEN ALGEBRAIC EXPRESSIONS

Directions: Remove common factors from the following expressions

1. $2x^3 + 3x^2 + x =$

2. $3x^4 + 6x^3 + 9x^2 =$

3. $9x^2y^3 + 6xy^4 + 3y^5 =$

4. $5mn^2 + 2m^2n + m^2n^3 =$

5. $7x^2y^3m^2 + 14xy^4m^3 + 7x^3y^3m^5 =$

6. $x^2y^3n^2 + x^4y^2m^3n^4 =$

7. $2x^2y^2 + 4x^3y + 6x^2y^4 =$

8. $x^5y^4 + x^6y^5 + x^3y^3 =$

9. $x^6m^4n^2 + x^7m^5n^3 + x^4m^3n^2 =$

10. $3x^3y^4 + 6x^2y^2 + 12x^3y^2 =$

SIMULTANEOUS EQUATIONS
UNIT VI

ASSIGNMENT SHEET #9--SOLVE THE SIMULTANEOUS EQUATION

Directions: Solve for both unknowns in the equations

1. $a + b = 15$
   $a - b = 5$

   $a =$ ____
   $b =$ ____

2. $3x + 2y = 26$
   $3x - 2y = 10$

   $x =$ ____
   $y =$ ____

3. $5a + 2b = 23$
   $a - 3b = 15$

   $a =$ ____
   $b =$ ____

4. $x - y = -4$
   $2x - 3y = -14$

   $x =$ ____
   $y =$ ____

5. $x + y = 5$
   $x - y = 1$

   $x =$ ____
   $y =$ ____

6. $a + b = 13$
   $a - b = 5$

   $a =$ ____
   $b =$ ____

7. $5a + 4b = 23$
   $5a - 4b = 7$

   $a =$ ____
   $b =$ ____

8. $5d - 3e = 5$
   $d - e = -1$

   $d =$ ____
   $e =$ ____

9. $x + y = 4$
   $3x - 2y = 7$

   $x =$ ____
   $y =$ ____

10. $x + 2y = 9$
    $x - 2y - 1$

    $x =$ ____
    $y =$ ____

11. $a + b = 25$
    $a - b = 5$

    $a =$ ____
    $b =$ ____

12. $9x - 5y = -13$
    $9x + 5y = 67$

    $x =$ ____
    $y =$ ____

13. $x + 4y = 7$
$2x - 4y = 6$

$x =$ ____
$y =$ ____

14. $2x + y = 7$
$3x + 2y = 12$

$x =$ ____
$y =$ ____

15. $3x + 2y = 19$
$3x - 2y = 11$

$x =$ ____
$y =$ ____

16. $a + b = 14$
$a - b = 6$

$a =$ ____
$b =$ ____

17. $3m + 8n = 58$
$3m - 8n = -22$

$m =$ ____
$n =$ ____

18. $x + y = 4$
$2x + 3y = 10$

$x =$ ____
$y =$ ____

19. $x + 2y = 7$
$3x + y = 16$

$x =$ ____
$y =$ ____

20. $x + 2y = 8$
$x - y = 5$

$x =$ ____
$y =$ ____

SIMULTANEOUS EQUATIONS
UNIT VI

ASSIGNMENT SHEET #10--WORD PROBLEMS FOR SIMULTANEOUS EQUATIONS

Directions: Solve the following problems.

1. The sum of two numbers is 13. Their difference is 1. What are the numbers?

2. The first of two numbers added to two times the second equals 9. The first number decreased by the second is 3. Find the numbers.

3. There are two numbers. Three times the first number increased by two times the second equals 28. Two times the first number increased by three times the second equals 27. Find the two numbers.

4. Find the numbers whose difference is 3 and, when the first number is decreased by three times the second number, the result is -11.

5. The difference of two numbers is -3. Twice the first number increased by three times the second is 29. Find the numbers.

6. Three times the length of a field increased by four times the width gives 24. Two times the length decreased by three times the width is -1. Find the length and width.

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SIMULTANEOUS EQUATIONS
UNIT VI

ANSWERS TO ASSIGNMENT SHEETS

ASSIGNMENT SHEET #1

1. $x(2x^2 + 3x + 1)$
2. $3y^3(3x^2 + 2xy + 3y^2$
3. $b^2(b^2 + b + 1)$
4. $b^2(b^2 + b + 1)$
5. $xy(x^3y$ $x - 2y)$
6. $m^2(m + 1 + 1)$
7. $m^2n(m^2n^2 + n + 1)$
8. $2x(3x + 2)$
9. $5b(b^2 + 2b + 3)$
10. $x^2bc(x^2b^2c + 1)$
11. $5 \cdot 3 \cdot a \cdot a \cdot b$
12. $3 \cdot 2 \cdot 3 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b$
13. $3 \cdot 3 \cdot 3 \cdot 2 \cdot x \cdot x \cdot x \cdot b \cdot b \cdot b$
14. $19 \cdot a \cdot a \cdot b \cdot b$

ASSIGNMENT SHEET #2

1. $(x + 2)(x - 2)$
2. $(bx^2 + 4)(b^2 - 4)$
3. $(3x + 5)(3x - 5)$
4. $(4 + 5c)(4 - 5c)$
5. $(5b^3 + 6)(5b^3 - 6)$
6. $(b + c)(b - c)$
7. $(2x^2 + 3b^2)(2x^2 - 3b^2)$

8. $(4x + 10)(4x - 10)$
9. $(8x^2 + 9)(8x^2 - 9)$
10. $(7m + 5)(7m - 5)$
11. $(12 + x)(12 - x)$
12. $(m + 13)(m - 13)$
13. $(b + 7)(b - 7)$
14. $(5x + 6c)(5x - 6c)$
15. $(L + 2)(L - 2)$
16. $(d^2 + 4)(d^2 - 4)$
17. $(x + 11)(x - 11)$
18. $(x^2 + 9)(x^2 - 9)$
19. $(10 + 4x)(10 - 4x)$
20. $(9 + 9x)(9 - 9x)$

ASSIGNMENT SHEET #3

1. $(x + 3)(x + 2)$
2. $(x - 3)(x - 2)$
3. $(x + 3)(x - 2)$
4. $(x - 3)(x + 2)$
5. $(x - 4)(x - 2)$
6. $(x + 4)(x + 3)$
7. $(x + 3)(x - 2)$
8. $(x - 3)(x + 2)$
9. $(x + 4)(x - 2)$
10. $(4x - 3)(4x - 1)$
11. $(3x + 2)(x - 1)$
12. $(3x - 1)(x + 5)$
13. $(5x + 1)(x - 5)$
14. $(3x - 5)(2x + 1)$
15. $(x - 7)(x - 1)$
16. $(x - 4)(x + 3)$

ASSIGNMENT SHEET #4

1. a^3
2. a^9
3. a^{m+n}

4. a^9
5. x^5
6. x^5
7. m^5
8. x^{m+a}
9. x^8
10. a^5
11. x^2
12. m
13. 1
14. x^4
15. b^4
16. c^2
17. m^{-2}
18. x^{-2}
19. a^{13}
20. L
21. a^{20}
22. a^6
23. x^{mn}
24. x^9
25. x^{10}
26. mn^2o^2
27. b^{16}
28. na^2
29. c^6n
30. Ln^2

31. x^{15}

32. x^{12}

ASSIGNMENT SHEET #5

1. $\sqrt{a}$
2. $\sqrt[3]{a27^2}$ or $\sqrt[3]{729}$
3. $\sqrt[3]{64}$
4. $\sqrt{36}$ or 6
5. $\sqrt{9}$ or 3
6. $\sqrt[3]{a}$
7. a
8. a^3
9. x^2
10. x^4
11. $\sqrt[3]{b}$
12. $\sqrt{w}$
13. $\sqrt[3]{z^2}$
14. $\sqrt[3]{a^4}$
15. $\sqrt{\frac{1}{-x}}$
16. $\sqrt[5]{p^2}$
17. $\sqrt[3]{\frac{1}{-r^2}}$
18. $\sqrt[5]{z^6}$
19. $\sqrt[3]{y^2}$
20. $\sqrt[4]{6w^3}$

ASSIGNMENT SHEET #6

1. 3
2. 4
3. 2
4. 3
5. $3x^2\sqrt{3}$
6. $3a^2\sqrt{11ab}$
7. $20x\sqrt{6y}$
8. $2xy^2\sqrt{3}$
9. $\dfrac{\sqrt{3}}{3}$
10. $\dfrac{\sqrt{ab}}{b}$
11. $\dfrac{\sqrt{x^3y}}{y}$
12. $\dfrac{8\sqrt{2}}{2}$ or $4\sqrt{2}$
13. $\dfrac{\sqrt{x}}{x}$
14. $\dfrac{3\sqrt{x}}{4\sqrt{yx}}$
15. $\sqrt{yx}$
16. $\dfrac{x\sqrt{y}}{y}$
17. $2x\sqrt{2y}$
18. $4ab^2c$

ASSIGNMENT SHEET #7

A. Add or subtract

1. $3\sqrt{3}$
2. $\sqrt{5}$
3. $3\sqrt{1/3}$
4. $10\sqrt{5}$
5. $38\sqrt{3}$
6. $2\sqrt{8x}$

B. Multiply and simplify

1. 3
2. $4\sqrt{5}$
3. 12
4. $160\sqrt{5}$
5. 40
6. $x - y$

7. $6xy\sqrt{2}$

8. $3\sqrt{8}$ or 2

C. Divide

1. $\sqrt{5}$

2. $\dfrac{3\,(3+\sqrt{2})}{7}$

3. $4(3-\sqrt{7})$

4. $\dfrac{3(1+\sqrt{3}}{2}$

5. $\dfrac{x+2\sqrt{x}+1}{x-1}$

6. $(\sqrt{2}+3)(\sqrt{3}-2)$

ASSIGNMENT SHEET #8

1. $x(2x^2+3x+1$
2. $3x^2(x^2+2x+3)$
3. $3xy^3(3x+2y+3y^2)$
4. $mn(5n+2m+mn^2)$
5. $7xy^3m^2(x+2ym+x^2n^3)$
6. $x^2y^2m^3n^2(ym+x^2n)$
7. $2x^2y(y+2y+3y^2)$
8. $x^3y^3(x^2y+x^3y^2+1)$
9. $x^4m^3n^2(y^2n+x^3m^2n+1)$
10. $3x^2y^2(xy^2+2+4x)$

ASSIGNMENT SHEET #9

1. a = 10
   b = 5
2. x = 6
   y = 4
3. a = 3
   b = 4
7. a = 3
   b = 2
8. d = 4
   e = 5
9. x = 3
   y = 1

4. x = 2
   y = 6

5. x = 3
   y = 2

6. a = 9
   b = 4

13. x = 5
    y = 1

14. x = 2
    y = 3

15. x = 5
    y = 2

16. a = 10
    b = 4

10. x = 5
    y = 2

11. a = 15
    b = 10

12. x = 3
    y = 8

17. m = 6
    n = 5

18. x = 2
    y = 2

19. x = 5
    y = 1

20. x = 6
    y = 1

ASSIGNMENT SHEET #10

1. x = 7 y = 6

2. x = 5 y = 2

3. x = 6 y = 5

4. x = 10 y = 4

5. x = 4 y = 7

6. x = 4 y = 3

SIMULTANEOUS EQUATIONS
UNIT VI

TEST

Directions: Solve the following problems as indicated.

1. A. Factor common monomial factors

   1. $x^4 + x^3 + x^2 =$

   2. $12x^4 + 9x^3 + 3x^2 =$

   3. $x^4y^2 + x^2y - 2xy^2 =$

   4. $m^4n^2 + 3m^4n^2 + 5m^4n^2 =$

   5. $a^4b^3c^2 + a^2b^4c^3 + a^5b^6c =$

   B. Factor the difference of two squares

   1. $a^2 + 5x + 6 =$

   2. $x^2 - 49 =$

   3. $x^6 - 81 =$

2. A. Simplify exponents and powers

   1. $a \cdot a \cdot a =$

   2. $a^2 \cdot a^2 \cdot =$

   3. $a^3 \cdot a^3 =$

   4. $\dfrac{a^4}{a^2} =$

   5. $\dfrac{a^5}{a^3} =$

   6. $\dfrac{a^2}{a^3} =$

7. $(x^3)2$ =

8. $(b^4)5$ =

9. $(c^3)5$ =

10. $81^{1/2}$ =

11. $8^{2/3}$ =

12. $a^{2/3}$ =

B. Express radicals in simplest forms

1. $\sqrt{9}$ =

2. $\sqrt{3\sqrt{27}}$ =

3. $\sqrt{a^4n^2}$

4. $\sqrt{\frac{a}{b}}$ =

5. $\sqrt{a^2 + b^2}$ =

6. $\frac{a}{\sqrt{b}}$ =

7. $\frac{\sqrt{a}}{\sqrt{b}}$ =

8. $\sqrt{20x^2b}$ =

9. $\sqrt{48a^2bc^2}$ =

C. Perform basic operations with radicals as the sign indicates

1. $8\sqrt{3} - 2\sqrt{3}$ =

2. $6\sqrt{5} - \sqrt{20}$ =

3. $2\sqrt{1/3} + 3\sqrt{1/3}$ =

4. $2\sqrt{y} \cdot 3\sqrt{y}$ =

5. $2\sqrt{y} \cdot \sqrt{3y} =$

6. $\sqrt{x^2y} \cdot \sqrt{xy} =$

7. $\dfrac{2\sqrt{5}}{\sqrt{4}} =$

8. $\dfrac{5\sqrt{2}}{\sqrt{3}} =$

9. $\dfrac{x\sqrt{b}}{\sqrt{4}} =$

3. A. Solve simultaneous equations. Solve for x and y.

1. $x + y = 18$
   $x - y = 2$

2. $x + y = 33$
   $x - y = 9$

3. $2x + 3y = 28$
   $2x - 3y = -8$

4. $3a + 8b = 58$
   $3a - 8b = -22$

5. $2x + 3y = 17$
   $3x - 2y = 6$

6. $3x + 2y = 13$
   $x + y = 5$

B. Solve word problems

1. Four times a number increased by a second number equals 29. Three times the first number decreased by the second equals 13. What are the numbers?

2. Twice a certain number added to a second number equals 27. The first number increased by twice the second equals 24. Find the numbers.

SIMULTANEOUS EQUATIONS
UNIT VI

ANSWERS TO TEST

1. A.
   1. $x^2(x^2 + x + 1)$
   2. $3x^2(4x^2 + 3x + 1)$
   3. $xy(x^3y + x - 2y)$
   4. $m^4n^2(1 + 3 + 5)$
   5. $a^4b^3c^2(b^2 + bc + a^3b^3)$

   B.
   1. $(b^2 + 4)(b^2 - 4)$
   2. $(x + 7)(x - 7)$
   3. $(x^3 + 9)(x^3 - 9)$

   C.
   1. $(a + 3)(a + 2)$
   2. $(x + 2)(x + 2)$
   3. $(x + 4)(x + 1)$

2. A.
   1. a^3
   2. a^4
   3. a^6
   4. a^2
   5. a^2
   6. a^{-1}
   7. x^6
   8. b^{20}
   9. c^{15}
   10. 9
   11. 4
   12. $\sqrt[3]{a^2}$

   E.
   1. 3
   2. 3
   3. a^2n
   4. $\frac{\sqrt{ab}}{b}$
   5. $a + b$
   6. $\frac{a\sqrt{b}}{b}$
   7. $\frac{\sqrt{ab}}{b}$
   8. $2x\sqrt{56}$
   9. $4ac\sqrt{3b}$

C.

1. $\sqrt{3}$

2. $4\sqrt{5}$

3. a^2

4. $6y$

5. $\frac{a\sqrt{b}}{b}$

6. $xy\sqrt{xy}$

7. $\sqrt{5}$

8. $\frac{5\sqrt{6}}{3}$

9. $4ac\sqrt{3b}$

3. A.

1. $x = 10$, $y = 8$

2. $x = 21$, $y = 12$

3. $x = 5$, $y = 6$

4. $a = 6$, $b = 5$

5. $x = 4$, $y = 3$

6. $x = 3$, $y = 2$

B.

1. $x = 6$, $y = 5$

2. $x = 10$, $y = 7$

RECTANGULAR COORDINATES/LINEAR EQUATIONS

UNIT VII

UNIT OBJECTIVE

After completion of this unit, the student should be able to solve rectangular coordinates and linear equations with given data. This knowledge will be evidenced by correctly performing the procedures outlined in the assignment sheets and by scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Illustrate linear functions graphically
   a. locate and plot points using rectangular coordinates
   b. graph linear equations
   c. solve systems of linear equations graphically
2. Solve linear equations by using algebra and determinants
   a. express an equation in graphic form
   b. algebraically solve two linear equations with two unknowns
   c. use determinates to solve two linear equations with two unknowns
   d. algebraically solve three linear equations with three unknowns
   e. using determinates, solve three linear equations with three unknowns

RECTANGULAR COORDINATES/LINEAR EQUATIONS

UNIT VII

SUGGESTED ACTIVITIES

I. Provide student with objective sheet.

II. Provide student with information and assignment sheets.

III. Discuss unit and specific objectives.

IV. Discuss information and assignment sheets.

V. Assist students as they work through assignment sheets.

VI. Give test.

VII. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective sheet

B. Information sheet

C. Assignment sheets-

1. Assignment Sheet #1--Locate and plot points

2. Assignment Sheet #2--Graph

3. Assignment Sheet #3--Solve equations graphically

4. Assignment Sheet #4--Express equations graphically

5. Assignment Sheet #5--Solve linear equaions algebraically

6. Assignment Sheet #6--Use determinate with two unknowns

7. Assignment Sheet #7--Algebraically solve three linear equations

8. Assignment Sheet #8--Use determinates to solve three linear equations in three unknowns

D. Answers to Assignment Sheets

E. Test

F. Answers to test

II. References

A. Chant, Alfred E., Barbara J. Montalto and Barbara A. Enfinger. Reaching Math Competence New York, NY: Globe Book Company, Inc., Cambridge Division, 1981.

B. Kerr, Donald J., Jr. Elementary Algebra New York, NY: McGraw-Hill, 1980.

C. Olivo, C. Thomas and Thomas P. Olivo. Basic Mathematics Simplified, 4th Edition. Albany, NY: Delmar Publishers Incorporated, 1977.

D. Shea, James T. Algebra Austin, TX: Steck-Vaughn Company, 1975.

E. Smith, Robert D. Vocational-Technical Mathematics Albany, NY: Delmar Publishers Incorporated, 1983.

F. Wiltsie, David. Practical Uses for Formulas and Equations Bishop, CA: Motivation Development Incorporated, 1982.

RECTANGULAR COORDINATES/LINEAR EQUATIONS

UNIT VII

INFORMATION SHEET

I. Terms and Definitions

A. Horizontal Axis or X-Axis--the line running horizontal across graph

(NOTE: Axes is plural for axis.)

B. Vertical Axis or Y-Axis--the middle line running vertical or perpendicular to graph

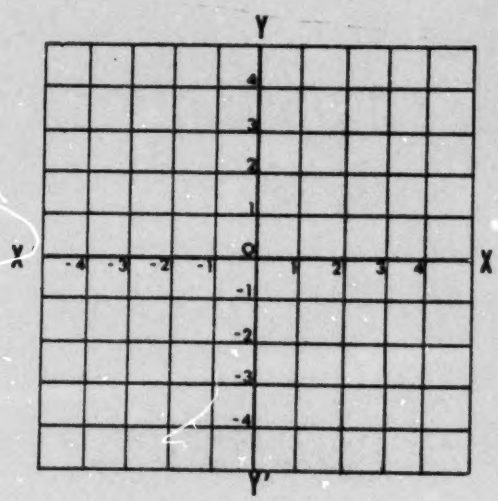
C. Abscissa--the X-value; always appears first

D. Ordinate--the Y-value

E. Coordinates--the abscissa and the ordinate make up the the coordinates

F. Quadrants--the four parts into which the coordinate axes divide a graph

II. Procedures



A. X axes are the horizontal lines going across the paper. Anything to the right of O (junction) is positive; anything to the left is negative

B. Y axes are the vertical lines; anything above the 0 (junction) is positive; anything below the 0 (junction) is negative

Example: Find coordinates of P

Step 1:

Find X first. It is on the second line; therefore, X coordinate = 2

Step 2:

Find Y; it is three up from X axis; therefore, Y = 3

Step 3:

Thus the coordinates = +2, +3

(NOTE: Always read the X axis first.)

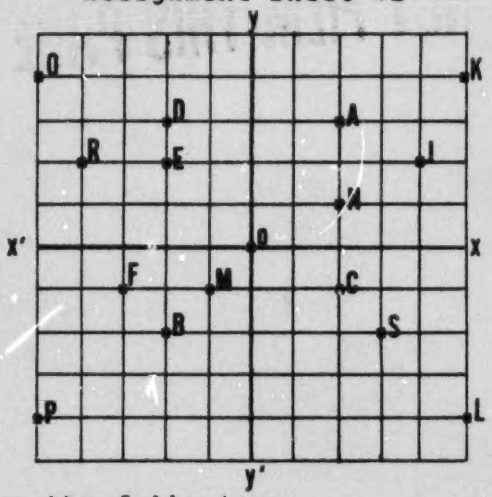
Example: What are coordinates of C:

1. X coordinate = -3 (to left of zero)
2. Y coordinate = +2 (above X axis)
3. Coordinates =(-3, +2)

RECTANGULAR COORDINATES/LINEAR EQUATIONS

UNIT VII

Assignment Sheet #2



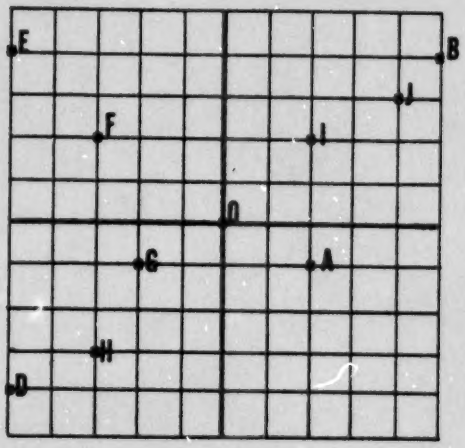
Give coordinates for the following:

1. A =
2. B =
3. C =
4. D =
5. E =
6. F =
7. G =
8. H =
9. L =
10. M =
11. N =
12. O =
13. P =
14. S =
15. O =

RECTANGULAR COORDINATES/LINEAR EQUATIONS

UNIT VII

TEST



1. Give coordinates for the following:

a. A =
b. B =
c. C =
d. D =
e. E =
f. F =
g. G =
h. H =
i. I =
j. J =

2. Plot the following on the above graph:

a. A = (-2,-2)
b. B = (-2,+2)
c. C = (+4,-3)
d. D = (+2,+3)
e. E = (+5,0)
f. F = (0,-3)
g. G = (0,+5)
h. H = (-1,+4)
i. I = (-1,+3)
j. J = (-4,-4)
k. K = (-5,-3)
l. L = (-5,+3)
m. M = (+1,+4)
n. N = (+1,+2)

QUADRATIC EQUATIONS

UNIT VIII

UNIT OBJECTIVE

After completion of this unit, the student should be able to solve quadratic equations. This knowledge will be evidenced by correctly performing the procedures outlined in the assignment sheets and by scoring 85 percent on the unit test.

SPECIFIC OBJECTIVES

After completion of this unit of instruction, the student will be able to:

1. Solve quadratic equations by factoring and check solutions.
2. Solve quadratic equations by completing the square and check solutions.
3. Solve quadratic equations by quadratic formula and check solutions.

QUADRATIC EQUATIONS

UNIT VIII

SUGGESTED ACTIVITIES

I. Provide student with objective sheet.

II. Provide student with information and assignment sheets.

III. Discuss unit and specific objectives.

IV. Discuss information and assignment sheets.

V. Assist students as they work through assignment sheets.

VI. Give test.

VII. Evaluate and record test results.

INSTRUCTIONAL MATERIALS

I. Included in the unit:

A. Objective sheet

B. Information sheet

C. Assignment sheets-

1. Assignment Sheet #1--Solve quadratic equations by factoring.

2. Assignment Sheet #2--Solve quadratic equations by completing the square.

3. Assignment Sheet #3--Solve quadratic equations by quadratic formula.

D. Answers to Assignment Sheets

E. Test

F. Answers to test

II. References

A. Chant, Alfred E., Barbara J. Montalto and Barbara A. Enfinger. Reaching Math Competence New York, NY: Globe Book Company, Inc., Cambridge Division, 1981.

B. Kerr, Donald J., Jr. Elementary Algebra New York, NY: McGraw-Hill, 1980.

C. Olivo, C. Thomas and Thomas P. Olivo. Basic Mathematics Simplified, 4th Edition. Albany, NY: Delmar Publishers Incorporated, 1977.

D. Shea, James T. Algebra Austin, TX: Steck-Vaughn Company, 1975.

E. Smith, Robert D. Vocational-Technical Mathematics Albany, NY: Delmar Publishers Incorporated, 1983.

F. Wiltsie, David. Practical Uses for Formulas and Equations Bishop, CA: Motivation Development Incorporated, 1982.

QUADRATIC EQUATIONS

UNIT VIII

INFORMATION SHEET

I. Terms and definitions

A. Factoring--the reverse of multiplication

B. Inspection--looking at problem and deciding on what the factors are

II. Procedures

A. Solving Quadratic Equations by Factoring--when solving a quadratic equation, the result of addition or subtraction is zero

Example: $x^2 + 16x + 64 = 0$

Solve by inspection factoring:

Step 1: x^2 factored will be (x)(x)
Step 2: 64 factored will be (8)(8)
Step 3: combined steps 1 & 2 (x 8)(x 8)
Step 4: choose correct signs (+,-) (x+8)(x+8)
Step 5: check

$$\begin{array}{r} x + 8 \\ x + 8 \\ \hline x^2 + 8x \\ 8x + 64 \\ \hline x^2 + 16x + 64 \end{array}$$

(same as above; therefore the solution is correct)

B. Solve Quadratic Equation by Completing the Square--when the equation is not a perfect square and thus cannot be factored, make it a perfect square to solve it.

Example:

$x^2 + 2x = 0$	1. Problem
$x^2 + 2x + 1 = 1$	2. Adding 1 to make square
$(x + 1)^2 = 1$	3. Factor
$x + 1 = \pm 1$	4. Extracting sq. root
$\therefore x = +1, -1,$ or 0	5. Using +1 root
$x = -1, -1,$ or -2	6. Using -1 root

When the coefficient of x^2 is 1, add the square of one-half the coefficient of x. Notice that the quantity on the right side of the equation might not be a perfect square, so leave the answer as a radical on the square root side.

Example:

$x^2 + 4x = 3$	1. Problem
$x^2 + 4x + 4 = 3+4$	2. Adding coefficient of x
$(x + 2)^2 = 7$	3. Factoring
$x - 2 = \pm\sqrt{7}$	
$x = 2 \pm\sqrt{7}$	4. Answer

C. Solving Quadratic Equations by using the Quadratic Formula--every quadratic equation can be written as

$ax^2 + by + c = 0$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: $x^2 + 7x + 6 = 0$

Procedure: a = number in front of $x^2 = 1$
b = number in front of $x = 7$
c = whole number = 6

thus, a = 1; b = 7; c = 6

Steps:

1. Write formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. Subtract numbers

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(6)}}{2(1)}$$

3. Multiply

$$x = \frac{-7 \pm \sqrt{49 - 24}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{25}}{2}$$

$$x = \frac{-7 \pm 5}{2}$$

4. Answer: $x = +6$ or -1

D. Check solutions by substitutions: After solving the problem, place the answer where the unknown was, work out the problem. The solution should be $0 = 0$.

Example: $x^2 + 2x = 0$
Answer is found to be -2.

$(-2)^2 + 2(-2) = 0$ Substitution
$4 - 4 = 0$
$0 = 0$ Checked solution.

QUADRATIC EQUATIONS

UNIT VIII

ASSIGNMENT SHEET #1--SOLVE QUADRATIC EQUATIONS BY FACTORING

Directions: Solve the following problems for the unknown.

1. $x^2 + 4x + 4 = 0$

2. $x^2 + 12x + 36 = 0$

3. $x^2 + 6x + 9 = 0$

4. $x^2 + 14x + 49 = 0$

5. $x^2 + 18x + 81 = 0$

6. $4x^2 + 12x = -9$

7. $9 + 12x + 4x^2 = 0$

8. $b^2 - 2b + 1 = 0$

9. $x^2 - 8x + 16 = 0$

10. $x^2 - 22x + 121 = 0$

QUADRATIC EQUATIONS

UNIT VIII

ASSIGNMENT SHEET #2--SOLVE QUADRATIC EQUATIONS COMPLETING THE SQUARE

Directions: Solve the following problems by making perfect square and checking.

1. $x^2 + 10x = 24$

2. $x^2 + 8x = 9$

3. $x^2 + 6x = -5$

4. $x^2 + 4x = 5$

5. $x^2 - 10x = -24$

6. $x^2 + 4x = 0$

7. $x^2 + 8x = 0$

8. $x^2 + 6x = 0$

9. $x^2 - 10x = 0$

10. $x^2 + 6x = 27$

11. $x^2 + 8x = 20$

12. $x^2 + 4x = 6$

13. $x^2 + 8x = 1$

14. $x^2 - 4x = 2$

QUADRATIC EQUATIONS

UNIT VIII

ASSIGNMENT SHEET #3--SOLVE QUADRATIC EQUATIONS USING QUADRATIC FORMULA

Directions: Solve the following problems for the unknown using the quadratic formula.

1. $x^2 + 5x + 6 = 0$

2. $x^2 + 5x + 2 = 0$

3. $4x^2 - 9x + 2 = 0$

4. $2y^2 - 7y + 3 = 0$

5. $2x^2 - x = 1$

6. $2x^2 - 8x + 3 = 0$

7. $2x^2 + 3x - 1 = 0$

8. $6x^2 = 5x + 3$

9. $3x^2 - 5x = -1$

10. $2x^2 + 5x = 1$

QUADRATIC EQUATIONS

UNIT VIII

ANSWERS TO ASSIGNMENT SHEETS

Assignment Sheet #1

1. $(x + 2)(x + 2)$ $x = -2$
2. $(x + 6)(x + 6)$ $x = -6$
3. $(x + 3)(x + 3)$ $x = -3$
4. $(x + 7)(x + 7)$ $x = -7$
5. $(x + 9)(x + 9)$ $x = -9$
6. $(2x + 3)(2x + 3)$ $x = -3/2$
7. $(3 + 2x)(3 + 2x)$ $x = -3/2$
8. $(b - 1)(b - 1)$ $b = 1$
9. $(x - 4)(x - 4)$ $x = 4$
10. $(x - 11)(x - 11)$ $x = 11$

Assignment Sheet #2

1. $x = -12, 2$
2. $x = -9, 1$
3. $x = -5, -1$
4. $x = -5, -1$
5. $x = 6, 4$
6. $x = -4, 0$
7. $x = -8, 0$
8. $x = -6, 0$
9. $x = 10, 0$
10. $x = 3, -9$
11. $x = 2, -10$
12. $x = -2, \pm\sqrt{10}$
13. $x = -4, \pm\sqrt{17}$
14. $x = 2, \pm\sqrt{6}$

Assignment Sheet #3

1. $x = -3, -2$
2. $x = -1/2, -2$
3. $x = 2, 1/4$
4. $x = 3, 1/2$
5. $x = 1/2, -1$
6. $x = \dfrac{4 \pm \sqrt{10}}{2}$
7. $x = \dfrac{-3 \pm \sqrt{17}}{4}$

8. $x = \frac{5 \pm \sqrt{97}}{12}$

9. $x = \frac{5 \pm \sqrt{13}}{6}$

10. $x = \frac{-5 \pm \sqrt{33}}{4}$

QUADRATIC EQUATIONS

UNIT VIII

TEST

1. Solve the following by factoring. Show your checking procedures.

   a. $x^2 - 4x + 3 = 0$

   b. $2x^2 - 3x - 2 = 0$

   c. $x^2 - 49 = 0$

2. Solve the following by completing the square and checking:

   a. $x^2 - 6x + 5 = 0$

   b. $2x^2 - 7x + 6 = 0$

   c. $2x^2 + x - 3 = 0$

3. Solve the following using the quadratic formula and check.

a. $x^2 - 6x = 2$

b. $x^2 - 12x = -6$

QUADRATIC EQUATIONS

UNIT VIII

ANSWERS TO TEST

1. a. $x = 1, 3$
   b. $x = -1/2, 2$
   c. $x = \pm 7$

2. a. $x = 5, 1$
   b. $x = -2, -3/2$
   c. $x = 3/2, 1$

3. a. $x = 3 \pm \sqrt{11}$

   b. $x = 6 \pm \sqrt{30}$